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MATHEMATICS

METRICAL CONTINGENTS

By F. I. SCHMIDOV

(Communicated by S. L. Soboleff, Member of the Academy, 11. I. 1943)

We shall examine arbitrary bounded point set in a three-dimensional Euclidean space. Definitions of the semi-tangent and contingent at a point to a given set have been given by Bouligand. The properties of sets in relation to the structure of their contingent have been investigated by Bouligand (1), Durand (2), Kolmogoroff, Schmidov and Verchenko (3).

As given in this paper, the definition of the contingent is based on the notion of set measure.

1. Let $\mu(E)$ denote a function of a set E. By $S(M, r)$ we denote a sphere of radius r with its centre at point M.

The point M is said to be one of positive lower density, if

$$\liminf_{r\to 0} \frac{\mu\{ES(M, r)\}}{r^{\tau}} > 0,$$

where $3 \geqslant \tau \geqslant 0$.

Denote by $K(\varphi, \xi, M)$ a half-cone having its apex at point M; φ is the angle at the apex, and ξ—the axis of the half-cone.

A ray ξ issuing from the point M is called metric semi-tangent at the point M to the set E, if

$$\limsup_{r\to 0} \frac{\mu\{EK(\varphi, \xi, M)S(M, r)\}}{r^{\tau}} > 0.$$

The sum of all metric semi-tangents at the point M to the set E is called a metric contingent.

2. It may easily be seen that the metric contingent is always a closed set. Indeed, let ξ be the limiting direction for the metric contingent at the point M. Then for a sequence of angles $\varphi_1 > \varphi_2 > \varphi_3 > \ldots > \varphi_n > \ldots \to 0$ we have a sequence of $\xi_1, \xi_2, \xi_3, \ldots, \xi_n, \ldots \to \xi$ belonging to the metric contingent, and for any angle $\varphi > 0$ there exists such a number N, that

$$\{K_n(\varphi_n, \xi_n, M)S_n(M, r)\} \subset \{K(\varphi, \xi, M)S(M, r)\}$$

for all $n \geqslant N$. Hence

$$\limsup_{r\to 0} \frac{\mu\{EK(\varphi, \xi, M)S(M, r)\}}{r^{\tau}} > 0,$$

and this means directly that ξ is an element of the metric contingent at the point M.

3. Denote by E_α a subset of all those points of a point set of positive lower density, for each of which, being given a certain number $\alpha > 0$, we can write

$$\liminf_{r\to 0} \frac{\mu\{ES(M, r)\}}{r^{\tau}} > \alpha. \qquad (1)$$

Then for the point $M \subset E_\alpha$ there exists such a positive number β that the inequality

$$\frac{\mu\{ES(M, r)\}}{r^\tau} > \alpha \tag{2}$$

holds for all positive $r \leqslant \beta$.

Let $E_{\alpha,\beta}$ denote a subset of all such points of the set E_α, at each of which the inequality (2) holds. Then using the method of Besikovich, we may establish the following

Theorem. *If a ray ξ issuing from the point $M \subset E_{\alpha,\beta}$ does not belong to the metric contingent at the point M to the set E, it neither belongs to Bouligand's contingent at the point M to the set $E_{\alpha,\beta}$.*

Proof. It follows from the closeness of the metric contingent that whenever ξ does not belong to the metric contingent at the point M to the set E, there exists such an angle $\varphi > 0$ that $K(\varphi, \xi, M)$ is free from the elements of the metric contingent of point M to set E. It is apparent that the theorem will be proved as soon as we have established that there exists a positive angle $\delta < \varphi$ such that $K(\delta, \xi, M)$ is free from points $M_j \subset E_{\alpha,\beta}$ having M for their limiting point, and this we are able to do.

Suppose, indeed, the reverse to be true and admit that $K(\delta, h, M)$ should, whatever the positive angle $\delta < \varphi$, contain an infinite sequence of points $M_j \subset E_{\alpha,\beta}$ tending to M. Let us be given an angle $\gamma > 0$ such that $K(2\gamma, h, M)$ lies inside $K(\varphi, \xi, M)$, where h is any generatrix of $K(\delta, \xi, M)$. Obviously

$$\gamma < \frac{\varphi - \delta}{2};$$

Now take a positive quantity ε satisfying the inequality

$$\left(\frac{1 + \sin\gamma}{\sin\gamma}\right)^\tau \varepsilon < \alpha; \tag{3}$$

Then, by the very definition of the metric semi-tangent, there exists such a positive number $\rho \leqslant \beta$, that for the point M the inequality

$$\frac{\mu\{EK(\varphi, \xi, M)\, S(M, r)\}}{r^\tau} < \varepsilon \tag{4}$$

holds with all positive $r \leqslant \rho$.

But in virtue of our assumption as to the presence of an infinite sequence of points $M_1, M_2, M_3, \ldots, M_j, \ldots$ of the set $E_{\alpha,\beta}$, tending to M inside $K(\delta, \xi, M)$, it is always possible, whatever the positive value of $r \leqslant \rho$, provided j is large enough, to form a sphere $S_j(M_j, r_j)$ lying entirely inside the surface of the half-cone $K(2\gamma, MM_j, M)$, this half-cone lying likewise entirely inside $K(\varphi, \xi, M)$.

By decreasing $r > 0$ one may make $S_j(M_j, r_j)$ touch the sphere $S(M, r)$ also. Further for the set $\{ES_j(M_j, r_j)\}$ we, according to (2), have

$$\frac{\mu\{ES_j(M_j, r_j)\}}{r_j^\tau} > \alpha, \tag{5}$$

and as $\{K(\varphi, \xi, M)\, S(M, r)\}$ contains $S_j(M_j, r_j)$, one may write

$$\frac{\mu\{EK(\varphi, \xi, M)\, S(M, r)\}}{r_j^\tau} > \alpha. \tag{6}$$

Now $r_j = r\,\frac{\sin\gamma}{1 + \sin\gamma}$. Substitution in (6) gives

$$\frac{\mu\{EK(\varphi, \xi, M)\, S(M, r)\}}{\left(r\,\frac{\sin\gamma}{1 + \sin\gamma}\right)^\tau} > \alpha. \tag{7}$$

From (4) and (7) follows

$$\left(\frac{1 + \sin\gamma}{\sin\gamma}\right)^\tau \varepsilon > \alpha. \tag{8}$$

Comparing this inequality with (3), we come to the conclusion that (4) is not consistent with (7).

4. Part of the results obtained for the contingent in the sense of Bouligand hold also for metric contingents. Thus, all theorems previously published in a paper by Schmidov and Verchenko ([3]) can be deduced from them.

Let us formulate the theorems for metric contingents.

Theorem 1. *A set E' of all those points of a point set of positive lower density, at each of which no whole straight line belongs to the metric contingent, is more than denumerable.*

Theorem 2. *A set E' of all those points of a point set of positive lower density, through each of which passes at least one plane such as has no whole straight line belonging to the metric contingent, is situated upon a denumerable number of unbent curves.*

Theorem 3. *A set E' of all those points of a point set of positive lower density, at each of which no space belongs entirely to the metric contingent, is situated on a denumerable set of surfaces satisfying the condition of Lipshitz.*

5. Let us express these results in terms of the theory of functions, where instead of $\mu(E)$ a linear measure of sets on the axis x will be considered.

Definition 1. A number λ is said to be the right (left) generalized derivative of $f(x)$ at point x_0, if for any quantity $\varepsilon>0$ the set of points x with

$$\lambda-\varepsilon<\frac{f(x)-f(x_0)}{x-x_0}<\lambda+\varepsilon$$

has at x_0 a right (left) positive upper density.

Definition 2. x_0 is said to be a point of positive density of a function $f(x)$, if for it there exists a number $\lambda<+\infty$ such that the set of points x with

$$\frac{f(x)-f(x_0)}{x-x_0}<\lambda$$

has a positive density at x_0.

From the theory of metric contingents as put forward here there immediately follows

Theorem. *At nearly every point of positive density x_0 of a function $f(x)$ either the one or the other of the two following cases takes place:*

1) *$f(x)$ has in x_0 an asymptotic derivative;*

2) *every number λ is both the right and left generalized derivative of $f(x)$ at the point x_0.*

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ASTRONOMY

ON THE MASS OF THE MOON'S ATMOSPHERE

By B. FESSENKOFF, Member of the Academy

It is generally assumed that our satellite is devoid of atmosphere. Various attempts to discover any traces of a gaseous envelope have failed to arrive at definite conclusions. It is certain that the mass of the lunar atmosphere, if any, must be very insignificant. What may be its upper limit? It may be shown that the polarization property of normally scattered light gives a very sensitive and simple means of determining the gaseous mass over a unit of the lunar surface.

Let us consider the feeble illumination of the lunar disc observed at the first or the last quarter very near to the terminator just outside of the illuminated part.

The brightness observed is evidently the sum of two very different illuminations. But it is mainly the effect of the halation due to the scattering of light in the Earth's atmosphere, as well as of the ashlight on the Moon. This component is not polarized at all. A secondary effect, probably contributing to the observed brightness, may be some scattering of light in the Moon atmosphere itself, which is fully polarized under the above-stated conditions. Consequently, in order to discover at a glance the presence or absence of atmosphere around our satellite, it is sufficient to rotate a polaroide before the objective of a telescope.

In a positive case the observed brightness must change more or less markedly. One may easily deduce its apparent degree of polarization and consequently the ratio n of the twilight intensity on the Moon to the halation produced by the Earth's atmosphere. It is necessary to find the scattering property μ of the lunar atmosphere.

It may be shown that

$$\mu = \frac{1}{2}\frac{(n-1)\,b}{Lm},$$

where L is the quantity of light from the Sun, falling normally at a unit surface, at the Moon distance; m, the mass of the atmosphere over a unit of lunar surface; b, the brightness of halation in an observed point of the disc.

Now it is necessary to determine the lunar halation effect which is very simple to find in relation to the solar one.

It is generally assumed that the halation, produced by some elementary surface ds at a distance l, is

$$dj = \frac{C\,ds}{l^2 + a^2},$$

a^2 being a certain constant. Accordingly, a finite surface produces

$$j = \iint \frac{C\,ds}{l^2 + a^2}.$$

This formula very satisfactorily represents the observations made by Dorno at Davos on the atmospheric brightness between the angular distances from 0°.1

to 10° from the border of the solar disc, assuming that

$$a^2 = 1.76.$$

This value of a^2 is characteristic of the Earth atmosphere. Observations show that apart from local irregularities the brightness over the lunar disc may be calculated by the well known Seeliger formula of diffuse reflexion

$$dj = \Gamma \frac{\cos i \cos \varepsilon d\sigma}{\cos i + \lambda \cos \varepsilon},$$

λ being nearly $^1/_2$.

In the case of full Moon $i = \varepsilon$, and the brightness over the lunar disc is constant, as is really the case.

Let us take the beginning of the rectangular coordinates in the centre of the lunar disc, the axis of abscissa being oriented along the equator of intensity. Then the apparent brightness at some point x, y is

$$J_{app} = \frac{x}{x + \lambda \sqrt{1 - r^2}}.$$

The distance r from the centre is expressed in terms of the lunar radius. Accordingly, the halation effect in the centre of the Moon at $\alpha = 90°$ is

$$j = \int\int \frac{J_{app}\, d\sigma}{l^2 + a^2}.$$

In this manner one can easily compare the halation brightness depending on the Moon at $\alpha = 90°$ in the centre of the lunar disc with the halation effect at some distance from the border of the full Moon. Now that the difference of stellar magnitudes of the Sun and the Moon is exactly known, one can calculate the ratio b/j_r (j_r being the Sun halation effect at a distance equal to r from the Sun); for example, at a distance of one solar radius from the Sun border we have

$$\frac{b}{j_r} = 1.824 \times 10^{-6}.$$

Now what remains is to compare photometrically the solar halation effect with the brightness of a standard surface, illuminated normally by direct solar rays.

Let K_r be the ratio of the halation brightness at the distance r to the brightness of the just mentioned standard surface. Then

$$K_r = \frac{j_r \cos \varepsilon}{\Gamma_1 L p^{\sec z f(\varepsilon)}},$$

L being the intensity of illumination at normal incidence without atmosphere; p, the coefficient of atmospheric transparence; Γ_1, the reflectivity of standard surface; ε, the angle of reflexion. The function $f(\varepsilon)$ represents the indicatrix of diffuse reflexion.

Summing up all relations stated above we have

$$\mu m = \frac{1}{2}(n-1)\, 1.824 \times 10^{-6} K_r \Gamma_1 p^{\sec z} \frac{f(\varepsilon)}{\cos \varepsilon}.$$

For the Earth atmosphere one can evidently take

$$\mu_1 M = -\frac{\log p}{4\pi \log e},$$

M being the mass of the atmosphere above a unit surface. It may reasonably be assumed that the scattering properties of the lunar and terrestrial atmospheres are nearly the same.

Accordingly

$$\frac{m}{M} = -\frac{4\pi \log e}{\log p} \frac{1}{2}(n-1)\, 1.824 \times 10^{-6} K_r \Gamma_1 p^{\sec z} \frac{f(\varepsilon)}{\cos \varepsilon}.$$

I examined the Moon at the last quarter through a polaroide and failed to discover any indication to the variation of brightness in the dark part near the terminator.

Accordingly, the value of $n-1$ is by no means greater than 4%.

Assuming

$$\Gamma_1 = \frac{0.8}{\pi}; \quad \sec z = 1.5; \quad p = 0.83; \quad f(\varepsilon) = \cos \varepsilon,$$

one can obtain

$$\frac{m}{m_1} = \frac{1}{2}(n-1)\,0.2 \times 10^{-4} K_1$$

or

$$m = m_1\, 0.4 \times 10^{-6} K_1.$$

Our extensive observations of the intensity of halation around the Sun furnish the value of K_1. This value is subject to some variations, but in the mean it may be assumed equal to 2 at a distance of one radius from the Sun border.

Consequently, the maximum mass of the atmosphere of the Moon as compared with that of the Earth is

$$m = m_1 \times 10^{-6}.$$

Therefore the method proposed to evaluate the mass of the lunar atmosphere is extremely sensitive. Careful photometrical observations of the Moon are very valuable to settle definitely the question about the presence of a gaseous envelope around our satellite.

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MATHEMATICAL PHYSICS

ON THE REPRESENTATION OF AN ARBITRARY FUNCTION BY AN INTEGRAL INVOLVING LEGENDRE'S FUNCTIONS WITH A COMPLEX INDEX

By V. A. FOCK, Member of the Academy

A good method for solving certain problems of the potential theory is to use toroidal coordinates ϑ, φ which are connected with the usual cylindrical coordinates $r = \sqrt{x^2+y^2}$ and z by means of the formulae

$$r = \frac{\operatorname{sh}\vartheta}{\operatorname{ch}\vartheta - \cos\varphi}; \qquad z = \frac{\sin\varphi}{\operatorname{ch}\vartheta - \cos\varphi}. \tag{1}$$

If the problem to be solved is one of axial symmetry, the potential Φ satisfying Laplace's equation in the physical space may be expressed, according to the formula

$$\Phi = \sqrt{2(\operatorname{ch}\vartheta - \cos\varphi)}\,\Psi, \tag{2}$$

by a function Ψ satisfying the equation

$$\frac{\partial^2\Psi}{\partial\vartheta^2} + \operatorname{cth}\vartheta\frac{\partial\Psi}{\partial\vartheta} + \frac{\partial^2\Psi}{\partial\varphi^2} + \frac{1}{4}\Psi = 0. \tag{3}$$

This equation admits of separation of variables. By putting

$$\Psi = p(\vartheta)\,s(\varphi) \tag{4}$$

one comes to the equations

$$\frac{d^2p}{d\vartheta^2} + \operatorname{cth}\vartheta\frac{dp}{d\vartheta} + \left(\mu^2 + \frac{1}{4}\right)p = 0, \tag{5}$$

$$\frac{d^2s}{d\varphi^2} - \mu^2 s = 0, \tag{6}$$

in which μ^2 is a parameter. If, by the nature of the problem, the range of variation of ϑ is $0 \leqslant \vartheta < \infty$, then the requirement that the solution be finite leads to positive values of the parameter μ^2. In the case of equation (5) the solution that meets this requirement is Legendre's function of the first kind with the complex index $\nu = i\mu - \frac{1}{2}$ and with the argument $\operatorname{ch}\vartheta$

$$p(\vartheta) = P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta), \tag{7}$$

while $s(\varphi)$ can be put equal to

$$s(\varphi) = a(\mu)\operatorname{ch}\mu\varphi + b(\mu)\operatorname{sh}\mu\varphi. \tag{8}$$

From the particular solutions of shape (4) a more general solution may be derived

$$\Psi = \int_0^\infty P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta)\{a(\mu)\operatorname{ch}\mu\varphi + b(\mu)\operatorname{sh}\mu\varphi\}\,d\mu. \tag{9}$$

The problem arises, To determine the functions $a(\mu)$ and $b(\mu)$ from some boundary conditions for Ψ. In case these conditions consist in that the function Ψ or a linear combination of Ψ and $\frac{\partial \Psi}{\partial \varphi}$ is given for two values of the coordinate φ, the problem evidently reduces to the determination of the function $f(\mu)$ from a given function $\psi(x)$, so as to have

$$\psi(x) = \int_0^\infty P_{i\mu-\frac{1}{2}}(x) f(\mu)\, d\mu \quad (1 \leqslant x < \infty). \tag{10}$$

In other words, the problem is reduced to the inversion of the integral (10) and to the expansion of an arbitrary function by Legendre's functions with a complex index. It is just this problem that we are to consider in the present article.

1. Some Properties of Legendre's Functions. Before we proceed to the solution of our problem, it is well to point out certain properties of Legendre's functions that will be used in the sequel. We define Legendre's functions of the first kind by means of Mehler's integral

$$P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) = \frac{2}{\pi}\int_0^\vartheta \frac{\cos \mu t\, dt}{\sqrt{2(\operatorname{ch}\vartheta - \operatorname{ch} t)}}. \tag{11}$$

Those of the second kind are then defined by means of the integral

$$Q_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) = \int_\vartheta^\infty \frac{e^{-i\mu t}\, dt}{\sqrt{2(\operatorname{ch} t - \operatorname{ch}\vartheta)}}. \tag{12}$$

The two kinds are connected by the relation

$$Q_{-i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) - Q_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) = i\pi \operatorname{tg}\mu\pi\, P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta). \tag{13}$$

From the last two formulae we have

$$P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) = \operatorname{cth}\mu\pi\, \frac{2}{\pi}\int_\vartheta^\infty \frac{\sin \mu t\, dt}{\sqrt{2(\operatorname{ch} t - \operatorname{ch}\vartheta)}}. \tag{14}$$

Substituting in the integral (11) the expansion

$$\sqrt{\frac{\operatorname{sh}\vartheta}{2(\operatorname{ch}\vartheta - \operatorname{ch} t)}} = \sqrt{\frac{\vartheta}{\vartheta^2 - t^2}}\left\{1 + \frac{1}{8\vartheta}\left(\operatorname{cth}\vartheta - \frac{1}{\vartheta}\right)(\vartheta^2 - t^2) + \ldots\right\} \tag{15}$$

which is a power series in $\vartheta^2 - t^2$, and integrating term by term we obtain for $P_{i\mu-\frac{1}{2}}$ an expression of the form

$$P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) = \sqrt{\frac{\vartheta}{\operatorname{sh}\vartheta}}\left\{J_0(\mu\vartheta) + \frac{1}{8\mu}\left(\operatorname{cth}\vartheta - \frac{1}{\vartheta}\right) J_1(\mu\vartheta) + \ldots\right\}. \tag{16}$$

Here the general term in brackets has the shape

$$A_n = a_n(\vartheta)\frac{J_n(\mu\vartheta)}{(\mu\vartheta)^n}, \tag{17}$$

where $a_n(\vartheta)$ is a polynomial in ϑ and $\operatorname{cth}\vartheta$, which with $\vartheta \to 0$ behaves like ϑ^{2n} and with $\vartheta \to \infty$ like ϑ^n. The series (16) converges when $\vartheta < \vartheta_0$ and diverges when $\vartheta > \vartheta_0$, where $\vartheta_0 = 2\pi\sqrt{2+\sqrt{5}}$. It may, however, if μ is great, be regarded as an asymptotic series fit to be used for all values of ϑ. The corresponding series for Legendre's functions of the second kind have the form

$$Q_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) \cong -\frac{\pi i}{2}\sqrt{\frac{\vartheta}{\operatorname{sh}\vartheta}}\left\{H_0^{(2)}(\mu\vartheta) + \frac{1}{8\mu}\left(\operatorname{cth}\vartheta - \frac{1}{\vartheta}\right) H_1^{(2)}(\mu\vartheta) + \ldots\right\} \tag{18}$$

$$Q_{-i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) \cong \frac{\pi i}{2}\sqrt{\frac{\vartheta}{\operatorname{sh}\vartheta}}\left\{H_0^{(1)}(\mu\vartheta) + \frac{1}{8\mu}\left(\operatorname{cth}\vartheta - \frac{1}{\vartheta}\right) H_1^{(1)}(\mu\vartheta) + \ldots\right\} \tag{19}$$

where $H_s^{(1)}$ and $H_s^{(2)}$ are Hankel's functions. These are always divergent series, but their coefficients are the same as in (16). Our series appear to be a novelty and are also applicable for complex value of μ. Their advantage over the ordinary asymptotic series is that when μ is large, they can also be used with small values of ϑ down to $\vartheta = 0$, while the ordinary series cannot.

The inequalities

$$\left| P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) \right| < P_{-\frac{1}{2}}(\operatorname{ch}\vartheta) < \frac{\vartheta}{2\operatorname{sh}\frac{\vartheta}{2}}, \tag{20}$$

$$\left| Q_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta) \right| < Q_{-\frac{1}{2}}(\operatorname{ch}\vartheta) < \frac{\pi}{2} \lg \operatorname{cth}\frac{\vartheta}{4}, \tag{21}$$

hold for real values of μ and ϑ [in (21) it is supposed that $\vartheta > 0$].

2. Inversion of Integral. Equation (10) may be rewritten thus

$$\psi(\operatorname{ch}\vartheta) = \int_0^\infty P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta)\, f(\mu)\, d\mu. \tag{22}$$

When $P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta)$ is replaced by its integral representation (11), the problem of finding the function $f(\mu)$ is reduced to the solution of an Abel's equation followed by the transformation of Fourier. In a purely formal way we obtain

$$f(\mu) = \mu \operatorname{th}\mu\pi \int_0^\infty P_{i\mu-\frac{1}{2}}(\operatorname{ch}\vartheta)\, \psi(\operatorname{ch}\vartheta) \operatorname{sh}\vartheta\, d\vartheta. \tag{23}$$

The formula gives the inversion of the integral (22).

The conditions under which (22) and (23) are valid can be formulated in the form of the two following theorems.

Theorem I. *If a function $\psi(x)$ given in the interval $(1 \leqslant x < \infty)$ is such that $\varphi(t) = 2\operatorname{sh}\frac{t}{2}\,\psi(\operatorname{ch} t)$ has its first derivative absolutely integrable over an infinite interval $(0 \leqslant t < \infty)$, while its second derivative is absolutely integrable over any finite interval, and if $\varphi(0) = 0$; $\varphi(\infty) = 0$, then $\psi(x)$ is representable in the form of the integral (10), where $f(\mu)$ is defined by*

$$f(\mu) = \mu \operatorname{th}\mu\pi \int_1^\infty P_{i\mu-\frac{1}{2}}(x)\, \psi(x)\, dx. \tag{24}$$

Theorem II. *If a function $f(\mu)$ is absolutely integrable over an infinite interval $(0 \leqslant \mu < \infty)$ and has its derivative absolutely integrable over any finite interval, and if $f(0) = 0$, then $f(\mu)$ is representable in the form of the integral (24), where $\psi(x)$ is defined by (10).*

The conditions of the first theorem may be extended and made less rigorous. In particular one can establish

Theorem III. *If a function $\psi(x)$, where $(1 \leqslant x < \infty)$, is such that there exists an integral*

$$\int_1^\infty \frac{|\psi(x)|\, dx}{\sqrt{x+1}}, \tag{25}$$

then for every point x, in whose neighbourhood $\psi(x)$ has a bounded variation, there holds

$$\frac{1}{2}[\psi(x+0) + \psi(x-0)] = \int_0^\infty P_{i\mu-\frac{1}{2}}(x)\, f(\mu)\, d\mu, \tag{26}$$

where $f(\mu)$ is defined by (24).

The way to prove Theorems I and II is by examining the course of computations which has led us to the inversion of the integral (10). Theorem III arises while the structure of formulae (10) and (24) is studied in detail, the formulae (16), (18) and (19) being made use of in this study.

Let us take an example before we finish. Put

$$\psi(x) = \frac{\sin \alpha}{[2(x + \cos \alpha)]^{3/2}} \qquad (-\pi < \alpha < \pi). \tag{27}$$

Then

$$f(\mu) = \mu \frac{\operatorname{sh} \mu\alpha}{\operatorname{ch} \mu\pi} \tag{28}$$

and we have

$$\frac{\sin \alpha}{[2(x + \cos \alpha)]^{3/2}} = \int_0^\infty P_{i\mu - \frac{1}{2}}(x) \frac{\operatorname{sh} \mu\alpha}{\operatorname{ch} \mu\pi} \mu \, d\mu \tag{29}$$

$$\frac{\operatorname{sh} \mu\alpha}{\operatorname{ch} \mu\pi} = \operatorname{th} \mu\pi \int_1^\infty P_{i\mu - \frac{1}{2}}(x) \frac{\sin \alpha}{[2(x + \cos \alpha)]^{3/2}} \, dx \tag{30}$$

which may easily be verified by direct computation.

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GEOPHYSICS

A HYDRODYNAMICAL THEORY OF PRESSURE AND TEMPERATURE WAVES AND OF CENTRES OF ATMOSPHERE ACTION

By E. N. BLINOVA

(Communicated by N. E. Kochin, Member of the Academy, 12. I. 1943)

The problem of long-range weather-forecasts by hydrodynamical methods has not been solved as yet. The reason is that the equations which describe the motion of the air atmosphere, surrounding the Earth as a spherical rotating body, are very complicate.

The author set before her the task to solve the problem of long-range forecasts of pressure, temperature and wind. With this aim in view she subjected the equations of motion to certain simplifications. The use of the latter made it possible to build a closed solution of the «stationary» problem consisting in the determination of the position and intensity of the centres of the atmosphere action. Applied to a concrete meteorological material, this solution revealed a picture of pressure distribution, which is in good agreement with the empirical data. The same simplifying assumptions made it possible easily to achieve the solution of the «non-stationary» problem concerning the determination for any given moment of the fields of pressure and temperature from their initial distribution.

1. Two forces acting upon the atmosphere are taken into account, that of gravity and the deviating force of the Earth' rotation. The Earth is taken to be a globe of a radius a.

As a starting equation use is made of Friedmann's equation, describing the change of a vertical vector in a given particle. The equation is written in the following shape:

$$\frac{d\Omega}{dt} - (\Omega, \nabla) V + \Omega \operatorname{div} V = -2 \operatorname{rot} [\omega, V] - \frac{1}{\rho T} [\operatorname{grad} T, \operatorname{grad} P], \qquad (1)$$

where P is pressure; ρ, density; T, temperature; V, velocity vector; Ω, vortex of the velocity vector; ω, vector of angular velocity of Earth rotation.

Let us introduce and examine the spherical coordinates: r, distance to the Earth centre; θ, latitude complement (increasing southwards) and λ, local latitude (increasing eastwards).

The motion of the atmosphere is taken as being composed of the zodiacal circulation and the perturbation of the latter. In other words, we are dealing with a motion, in which the velocity components along the axes (θ) and (λ) and the pressure are represented in the following form:

$$V_\theta = v_\theta(\theta, \lambda, t),\ V_\lambda = \bar{v}_\lambda(\theta) + v_\lambda(\theta, \lambda, t),\ P = \bar{p}(\theta) + p(\theta, \lambda, t),^* \qquad (2)$$

where t is time. The perturbation elements v_θ, v_λ, p are supposed to be small. Let us put, together with Haurwitz ([1]):

$$\bar{v}_\lambda = \alpha a \sin\theta, \qquad (3)$$

* Here, as well as in the sequel, the mean values refer to altitude.

where α is the angular velocity of air motion with respect to the Earth (independent of θ). In this case $\bar{p}$ is determined from the formula:

$$\bar{p} = p_0 + \rho a^2 \alpha \omega \sin^2 \theta \tag{4}$$

(the condition of geostrophic wind is assumed).

For the temperature we take

$$T = \bar{T}(\theta) + T'(\theta, \lambda) + T''(\theta, \lambda, t), \tag{5}$$

where $\bar{T} + T'$ is the mean monthly (or seasonal) temperature distribution due to insolation, turbulence and to the presence of continents and oceans, which is supposed to be known. T'' is the deviation from this temperature distribution.

$$\bar{T}(\theta) = T_0 + M \sin \theta \tag{6}$$

(T_0 and M are constants). T' and T'' are assumed small.

The equation of continuity makes it possible to introduce the function of current ψ:

$$v_\theta = -\frac{\partial \psi}{a \sin \theta \partial \lambda}, \quad v_\lambda = \frac{\partial \psi}{a \partial \theta}, \tag{7}$$

which can conveniently be represented in the form of

$$\psi(\theta, \lambda, t) = \psi'(\theta, \lambda) + \psi''(\theta, \lambda, t), \tag{8}$$

where the part of ψ' which does not depend on time is determined by the rpesence of the term T' in the expression (5).

Projecting (1) upon the axis (r), neglecting the terms with vertical velocity, making use of equality (7) and discarding the terms of the second order of smallness, we obtain:

$$\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right)\left[\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 \psi}{\partial \lambda^2}\right] + 2(\alpha + \omega)\sin\theta \frac{\partial \psi}{\partial \lambda} = \frac{1}{\rho \bar{T}}\left[\frac{d\bar{p}}{d\theta} \cdot \frac{\partial (T' + T'')}{\partial \lambda} - \frac{d\bar{T}}{d\theta}\frac{\partial p}{\partial \lambda}\right]. \tag{9}$$

The equation of motion will be written in the following shape:

$$\frac{\partial p}{\rho \sin \theta \partial \lambda} = -\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right)\frac{\partial \psi}{\partial \theta} + 2(\alpha + \omega)\operatorname{ctg}\theta \frac{\partial \psi}{\partial \lambda}. \tag{10}$$

It will now be convenient to search for p in the form of

$$p(\theta, \lambda, t) = p'(\theta, \lambda) + p''(\theta, \lambda, t) \tag{11}$$

where the term p' is determined by the presence of the term ψ' in ψ.

The equation of heat inflow yields:

$$\frac{\partial T''}{\partial t} + \alpha \frac{\partial T''}{\partial \lambda} = \frac{1}{a^2 \sin\theta}\frac{d\bar{T}}{d\theta}\frac{\partial \psi''}{\partial \lambda}. \tag{12}$$

By solving the system of equations (9), (10), (12), we determine the unknown functions ψ, p, T''.

2. The solution of the problem is essentially simplified if in (9) we neglect T'' as compared to T', and the term $\frac{d\bar{T}}{d\theta}\frac{\partial p}{\partial \lambda}$ as compared to $\frac{d\bar{p}}{d\theta}\frac{\partial T'}{\partial \lambda}$. That we have a right to do so has been proved by the numerical calculations performed by the author.

In this case equation (9) will assume the following shape:

$$\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right)\left[\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 \psi}{\partial \lambda^2}\right] + 2(\alpha + \omega)\sin\theta \frac{\partial \psi}{\partial \lambda} = a^2 \alpha \omega \frac{\sin 2\theta}{\bar{T}}\frac{\partial T'}{\partial \lambda}. \tag{13}$$

The stationary part of the solution of equation (13) (ψ') is sought for in the form of a series of spherical functions:

$$\psi'(\theta, \lambda) = \sum_{n=1}^{\infty} \sum_{m=1}^{n} (C_n^m \cos m\lambda + C'^m_n \sin m\lambda) P_n^m (\cos\theta). \tag{14}$$

In this way for the coefficients C_n^m, C'^m_n the following expressions are obtained:

$$C_n^m = \frac{2a^2\omega}{\dots} \frac{\frac{n-m}{2n-1}\delta^m_{n-1} + \frac{m+n+1}{2n+3}\delta^m_{n+1}}{\frac{2(\alpha+\omega)}{\alpha} - n(n+1)},$$

$$C'^m_n = \frac{2a^2\omega}{\tilde{T}} \frac{\frac{n-m}{2n-1}\delta'^m_{n-1} + \frac{m+n+1}{2n+3}\delta'^m_{n+1}}{\frac{2(\alpha+\omega)}{\alpha} - n(n+1)} \tag{15}$$

where δ_n^m, δ'^m_n are the coefficients of expansion of T' into a series of spherical functions; $\tilde{T}$, the mean value of temperature for the whole atmosphere *.

Pressure $p'(\theta, \lambda)$ which corresponds to this stationary term of the current function is found from the equation of motion (10):

$$p'(\theta, \lambda) = 2\rho(\alpha+\omega) \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left\{ \left[C^m_{n-1} \frac{(n-m)(n-1)}{(2n-1)n} + C^m_{n+1} \frac{(n+2)(n+m+1)}{(n+1)(2n+3)} \right] \cos m\lambda + \right.$$
$$\left. + \left[C'^m_{n-1} \frac{(n-m)(n-1)}{(2n-1)} + C'^m_{n+1} \frac{(n+2)(n+m+1)}{(n+1)(2n+3)} \right] \sin m\lambda \right\} P_n^m(\cos\theta). \tag{16}$$

In the search for the non-stationary part of the solution of equation (13), (ψ''), we obtain:

$$\psi''(\theta, \lambda, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{n} [D_n^m \cos(m\lambda + \sigma t) + D'^m_n \sin(m\lambda + \sigma t)] P_n^m(\cos\theta), \tag{17}$$

where

$$\sigma = \frac{2(\alpha+\omega)m}{n(n+1)} - \alpha m, \tag{18}$$

while D_n^m, D'^m_n are the unknown coefficients to be determined. For the pressure $p''(\theta, \lambda, t)$ which corresponds to the non-stationary part of the current function, we find from (10) an expression analogous to that for p', where $m\lambda$ is substituted for $m\lambda + \sigma t$, C_n^m for D_n^m, C'^m_n for D'^m_n.

The non-stationary part of pressure as represented above is used by us in order to find the coefficients D_n^m, D'^m_n. By doing so we arrive at the systems of equations

$$A_n^m - 2\rho(\alpha+\omega)\left[D^m_{n-1} \frac{(n-m)(n-1)}{(2n-1)n} + D^m_{n+1} \frac{(n+2)(n+m+1)}{(n+1)(2n+3)} \right] = 0$$
$$A'^m_n - 2\rho(\alpha+\omega)\left[D'^m_{n-1} \frac{(n-m)(n-1)}{(2n-1)n} + D'^m_{n+1} \frac{(n+2)(n+m+1)}{(n+1)(2n+3)} \right] = 0 \tag{19}$$

where A_n^m, A'^m_n are the coefficients of the series of spherical functions into which the expression $P|_{t=0} - (\bar{p} + p')$ can be expanded. Let us search a solution symmetrical with respect to the equator. In this case it is expedient

* The denominators for C_n^m, C'^m_n may prove close to zero for certain values of n. In this case a kind of peculiar resonance is obtained. The possibility of a similar effect has for the first time been suggested by Haurwitz (1). If, however, the term containing $\frac{dT}{d\theta}\frac{\partial p}{\partial \lambda}$ is not discarded on the right-hand side of equation (9), then systems of equations will serve for the determination of C_n^m, C'^m_n. The resonance in this case is smoothed out.

to take in (16) and (19) all the n of the same evenness as m. Equations (19) will make it then possible to determine all the coefficients D_n^m, D'^m_n successively.

Formula (16) describes the position and intensity of the centres of action of the atmosphere. The expression for $p''(\theta, \lambda, t)$ enables us to find for any moment the field of pressure from its given initial distribution.

After we have found the expression for the non-stationary term of the current function (ψ''), the temperature deviation (T'') from its mean monthly (or seasonal) distribution ($\overline{T}+T'$) may easily be found from equation (12). In this way we obtain:

$$T''(\theta, \lambda, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{n} \{[B_n^m \cos m(\lambda - \alpha t) + B'^m_n \sin m(\lambda - \alpha t)] P_n^m - \\ - 2N_n \left[D_n^m \sin\left(m\lambda + \frac{\sigma - m\alpha}{2} t\right) - D'^m_n \cos\left(m\lambda + \frac{\sigma - m\alpha}{2} t\right) \right] \times \\ \times [(n+m) P_{n-1}^m + (n-m+1) P_{n+1}^m] \sin \frac{\sigma + m\alpha}{2} t \Big\}, \qquad (20)$$

where $N_n = \frac{M}{a^2} \frac{n(n+1)}{(2n+1)(\omega+\alpha)}$, B_n^m and B'^m_n are the coefficients of the series of spherical functions obtained by expanding the expression $T|_{t=0} - (\overline{T}+T')$.

3. Formula (16) has been used in order to obtain theoretically the mean pressure distribution during January over the northern hemisphere. To construct a series for $\overline{T}+T'$ use was made of the world's map of mean air temperatures during January (reduced to the sea-level), as given in the Great Soviet Atlas of the World. α has been put equal to 0.042 ω.

From the formulae (15) $\left(\text{with } a = 6.37 \times 10^6\, m, \overline{T} = 255°\text{abs.}, \omega = 7.29 \times \right.$ $\left. \times 10^{-5} \frac{1}{\text{sec.}} \right)$ there were calculated the values of coefficients C_n^m, C'^m_n; after this the series for $\overline{p}(\theta) + p'(\theta, \lambda)$ was constructed. This series served us to construct the January fields of pressure at an altitude of 4000 m and at sea-level. Thus, the Iceland and Aleutic minima, as well as the maxima for the Pacific, Siberia and Canada have been obtained.

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CHEMISTRY

IS QUENCHING NECESSARY IN THE THERMAL CRACKING OF METHANE TO ACETYLENE?

By N. I. KOBOZEV

(Communicated by A. E. Arbusov, Member of the Academy, 4. I. 1943)

It has become a tradition to think that in every high temperature process the «quenching» (chilling) of the endothermal product through a rapid cooling of the reaction mixture is useful and even necessary. Such a view is also accepted in respect to the cracking of methane to acetylene. By the requirement that cooling be rapid considerable difficulties are introduced into the practical side of the process, and the energy of the reaction gases can be utilized only to a very limited extent. But whether the «quenching» is at all necessary, is in doubt. In any case the idea of its usefulness is more based upon indirect kinetic considerations, than direct experiments.

In our laboratory comparative tests have been made on the influence of rapid and slow cooling of the reaction gases upon the yields of acetylene and ethylene in the cracking of methane containing gases. The experiments were carried on in quartz, porcelain and carbon tubes in a platinum and in a Tammann stove at temperatures varying from 990 to 2000° C.

The tubes used for the cracking process were of such length, that after its exit from the stove the gas passed along the tube for another 70 cm. In some experiments the outside end of the tube was strongly cooled by water, in others it was protected from cold by a thick layer of asbestos.

The results of the experiments are given in the table.

D denotes the «after-cracking» in per cent, or the percentage of the increase or decrease of unsaturated compounds in insulation and in cooling tests.

$$D_{(C_2H_4 \text{ or } C_2H_2)} = \left[\frac{(C_2H_4 \text{ or } C_2H_2) \text{ with insulation}}{(C_2H_4 \text{ or } C_2H_2) \text{ with cooling}} - 1\right] 100;$$

$$D_{(C_2H_2+C_2H_4)} = \left[\frac{(C_2H_2 + C_2H_4) \text{ with insulation}}{(C_2H_2 + C_2H_4) \text{ with cooling}} - 1\right] 100.$$

The data obtained show that when applied to the cracking of methane quenching is not only unnecessary, but, as a rule, is even harmful. It turns out, indeed, to be more expedient to allow the gas the possibility of «after-cracking» after it has passed the heating zone. In this way one may increase the yield of unsaturates (and, thereby, cut down the expense of energy) to 40%. In one test only (No. 13) cooling proved useful, owing, probably, to a low rate of gas movement not consistent with the high temperature of the cracking process.

It is noteworthy that the substitution of air cooling for special water cooling (in tests 9* and 9) had no influence whatever upon the total yield of unsaturates (10.8% and 11.1%), but merely changed the ethylene—acetylene ratio in favour of the latter.

The possibility of carrying on the reaction beyond the heating zone with a gain rather than loss in the reaction products is, no doubt, due to the non-equilibrium

No. of run	Character of run	t° of run in °C	Velocity, $\frac{\text{litre}}{\text{min.}}$	Expansion	P. c. in final gas			Reduction p. c.			Relative yield of C_2H_2, γ	$D_{C_2H_2}$ %	$D_{C_2H_4}$ %	$D_{(C_2H_2+C_2H_4)}$ %
					C_2H_2	C_2H_4	$C_2H_2+C_2H_4$	C_2H_2	C_2H_4	$C_2H_2+C_2H_4$				
A. Runs in Quartz and Porcelain Tubes														
2	Cooling	990	2.0	—	1.5	8.7	10.2	—	—	—	0.15	+93	−17	−1
2a	Insulation	990	2.0	—	2.9	7.2	10.1	—	—	—	0.29			
3	Cooling	1 050	2.7	—	1.3	8.9	10.2	—	—	—	0.13	+62	0	+7.9
3a	Insulation	1 050	2.7	—	2.1	8.9	11.0	—	—	—	0.19			
4	Cooling	1 080	4.0	—	1.0	8.3	9.3	—	—	—	0.11	+80	+13	+20
4a	Insulation	1 080	4.0	—	1.8	9.4	11.2	—	—	—	0.16			
5	Cooling	1 090	4.3	1.153	1.0	8.0	9.0	1.15	9.22	10.37	0.11	+40	+19	+25
5a	Insulation	1 100	4.3	1.160	1.4	8.5	10.0	1.62	11.02	12.64	0.13			
6	Cooling	1 140	—	—	—	—	5.3	—	—	—	—	—	—	+29
6a	Insulation	1 140	—	—	—	—	6.8	—	—	—	—			
7	Cooling	1 255	8.5	—	0.3	3.9	4.2	—	—	—	0.17	+100	+28	+33
7a	Insulation	1 230	8.5	—	0.6	5.0	5.6	—	—	—	0.11			
8	Cooling	1 270	2.0	—	3.9	0.7	4.6	—	—	—	0.85	+31	+86	+39
8a	Insulation	1 270	2.25	—	5.1	1.3	6.4	—	—	—	0.80			
B. Runs in Carbon Tubes														
9*	Without cooling	1 640	15.6	1.40	7.0	0.7	7.7	9.8	1.0	10.8	0.91	+49	−80	−3
9	Cooling	1 610	16.1	1.408	4.7	3.2	7.9	6.62	4.5	11.1	0.60	+36	+27	+32
9a	Insulation	1 470	14.0	1.55	5.8	3.7	9.5	9.0	5.7	14.7	0.61			
10	Cooling	1 418	24.0	—	3.4	4.0	7.4	—	—	—	0.46	+20	+25	+22
10a	Insulation	1 390	24.0	—	4.0	5.0	9.0	—	—	—	0.44			
11	Cooling	1 596	29.0	1.18	3.5	2.3	5.8	4.1	2.7	6.8	0.60	+30	+18	+26
11a	Insulation	1 625	29.0	1.20	4.5	2.7	7.2	5.4	3.2	8.6	0.62			
12	Cooling	1 800	29.0	1.28	3.5	2.6	6.1	4.5	3.3	7.8	0.57	+36	+46	+40
12a	Insulation	1 800	29.0	1.35	4.5	3.6	8.1	6.1	4.9	11.0	0.56			
13	Cooling	2 000	21.0	1.536	7.3	2.9	10.2	11.2	4.4	15.6	0.72	+16	−33	−27
13a	Insulation	2 000	20.0	1.650	5.7	1.2	6.9	9.4	2.0	11.4	0.82			

Notes. 1. Gas composition in runs No. 2, 2a, 3, 3a, 4, 4a, 5, 5a: $\frac{CH_4\ C_2H_6\ H_2\ N_2}{62\ 13.5\ 21.5\ 3\%}$; in other runs: $\frac{CH_4\ C_2H_6\ H_2\ N_2}{65\ 8.5\ 22\ 4.5\%}$.

2. In runs, where expansion was determined, after-cracking was calculated using reduced p. c. of C_2H_2 and C_2H_4; in other runs non-reduced values were used.

character of the «cracking process»; in this it differs from such reactions as the formation of NO and HCN at high temperatures.

The continuation of cracking, after the heating zone is reached, that is in an isolated adiabatic zone, leads to an autocooling of the reaction gases, because the formation of C_2H_4 and C_2H_2 is connected with a considerable absorption of heat, and, simultaneously, to an autoinhibition of the process (the adiabatic autoinhibition is also a distinguishing feature of the cracking of methane as compared to equilibrium reactions). Supposing the cooling of the gases in the zone of «adiabatic after-cracking» to proceed entirely at the expense of the formation of unsaturates, the magnitude of after-cracking may be defined (in per cent) as

$$D_{(C_2H_2+C_2H_4)\%} = \frac{E_T}{E_r} \cdot 100 \frac{t-t_d}{t-t_0}, \text{ or } = D_0 \frac{t-t_d}{t-t_0}, \text{ where } D_0 = \frac{E_T}{E_r} \cdot 100,$$

where t is the cracking temperature (° C); t_d is the lower limit of the temperature of «after-cracking», to which the gas is autocooled; E_T is the heat content of the reaction gases issuing from the cracking zone (from t_0 to t) per unit (1 m³) of unsaturates; E_r is the heat absorbed in the formation of a unit of unsaturates.

From this equation D_0 and t_d can be found.

If the temperature range of the experiment is 990—1270° C, then $D_0 \simeq 200$ and $t_d \simeq 990°$ C; this temperature is actually very near the initial temperature of cracking determined directly, and, what is more, the value for $^1/_{100}\, D_0 \simeq 2.0$ agrees well with the magnitude of $\frac{E_T}{E_r}$ which is about 2.4 on the average.

This concurrence of values shows that at average temperatures (up to 1300° C) the retardation of the reaction in the zone of after-cracking is almost entirely (up to $80\% \simeq 100 \frac{2.0}{2.4}$) due to the «adiabatic autocooling» of the gases as a result of the formation of an additional amount of unsaturated hydrocarbons. At higher temperatures the external loss of heat is greater: $D_0 = 110$, $t_d = 1150°$ C, and the gas is autocooled on account of the reaction to about 50%.

As may be seen from these figures, the «adiabatic autocooling» of the reaction gases by endothermal processes can greatly lower the temperature (by 300—400° C) and considerably increase the yield (up to 40%).

Probably, the wide discrepancy observed between the expenditure of energy in the production of acetylene, as given usually in patents, and the figures obtained by thermodynamic calculations is directly connected with the phenomenon of such an «adiabatic after-cracking» [see (¹), equations (14—17)].

The use of our data and of equations (5—9) from (¹) gives the following indices for this process: total cracking $\Delta \simeq 70\%$; useful cracking $\sigma \simeq 70\%$; expansion $(1+\alpha) \simeq 1.35$; reduced % of $C_2H_2 = 12\%$. From this the heat of the reaction is $E_r \simeq 5.7$ kWhr/m³ C_2H_2 and only $E_T \simeq 4$ kWhr/m³ C_2H_2 for the energy expended in heating the gases to the temperature of the arc (allowing for a 20% loss of energy in the arc through radiation*).

Putting the temperature of the electric arc at about 2600—2900° C (in accordance with direct measurements and calculations (²) and also with the equilibrium yields of NO and HCN in these arcs in the reaction $N_2 + O_2$ and $CH_4 + N_2$), the necessary energy of the heating of the gas will be 17.6 kWhr/m³C_2H_2 (instead of 4 kWhr/m³C_2H_2 according to patent data!), and the total expenditure of energy $E_t = 23.3$ kWhr/m³C_2H_2. This figure agrees with laboratory data on high temperature cracking [see Table 1, runs No. 3—8 (³)], but is twice the patent values**.

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* By analogy with technical electrical arcs for the oxidation of nitrogen.

** In general one should note (Table 1) that no one has been able under laboratory conditions to obtain acetylene by the thermal cracking of methane at atmospheric pressure without using either higher homologues or vacuum, whereby the expense of energy is considerably lessened with an expenditure in energy below 17 kWhr/m³ C_2H_2 (taking into account a 20% loss of heat by radiation). We do not think that on this account patent values should give ground to doubt, yet the reason for such a discrepancy should be looked for persistently.

To a low value of $E_T \simeq 4$ kWhr/m³ C_2H_2 there naturally should correspond a narrow temperature interval of the heating of the entering gases. Therefore in order to obtain agreement with practical (patent) values, one should either accept a low temperature for the electric arc or considerable heating of the entering gases at the expense of reaction gases. But this leads to highly improbable results: if the gas is taken without preheating, the temperature of the electric arc will be only about 850° C (?!), and if a normal temperature of the arc is accepted (2900° C), the temperature of preheating is calculated as 2500° C (?!), both cases being, evidently, impossible.

Formally a balance can be drawn if one freely puts the temperature of the arc at about 1600° C and that of the preheating of the gas at 700° C. This, however, implies so complete a heat exchange between the entering and the reaction gases (75%), that very little reliance can be placed upon this version, too. One has to recognize that so far there is no satisfactory explanation for these discrepancies*.

According to the author, such a sharp contradiction between practice and thermodynamics (1st principle) cannot be explained away by commonplace reasons. The existence of some special mechanism of energy compensation should be assumed (either laid down consciously or materialized automatically in the cracking in an electric arc). This mechanism is probably acting in the after-reaction stage of the process and bearing upon the utilization of outlet gas energy. At present there are no experimental data that can prove the existence of such a mechanism. However, it seems probable that it should be connected with the phenomenon of «adiabatic after-cracking» and «adiabatic autoinhibiting» of the process beyond the pyrolysis zone. Indeed, the exterior rapid retardation of the reaction with an inevitable loss of heat of high potential has been proved by us not to be necessary, and even, on the contrary, the possibility and usefulness of a continuation of the reaction beyond the heating zone has been demonstrated; this extends and varies considerably the possibility of using the energy of the outlet gases and is thus a help in the search of the compensation energy mechanism in the cracking of methane gases to acetylene**.

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* These discrepancies persist irrespective of the inner mechanism of cracking in the electric arc (electrical or thermal nature of the reaction), inasmuch as all the energy of the arc is finally converted into heat.

** The problem of the existence of such a mechanism is one of the competition capacity of cracking with the carbide process. At least the energetic parity of these methods is needed, i. e. actually the expense of energy in the cracking process should reach at the level of patent values: 10—13 kWhr/m³ C_2H_2.

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CHEMISTRY

POLYTHERM OF AMMONIUM CHLORIDE—AMMONIUM NITRATE—WATER TERNARY SYSTEM

By B. A. PAVLOV, N. A. BUTOVICH and A. G. BERGMANN

(Communicated by V. G. Chlopin, Member of the Academy, 16. XII. 1942)

The ternary system investigated is of no minor practical import. It is a part of the system of a compound fertilizer of the type K', NH'_4/NO'_3, $H_2PO'_4$, Cl'.

In studying it the visual polythermal method was employed. The polythermal diagram of solubility was studied on sections through the system. We started from

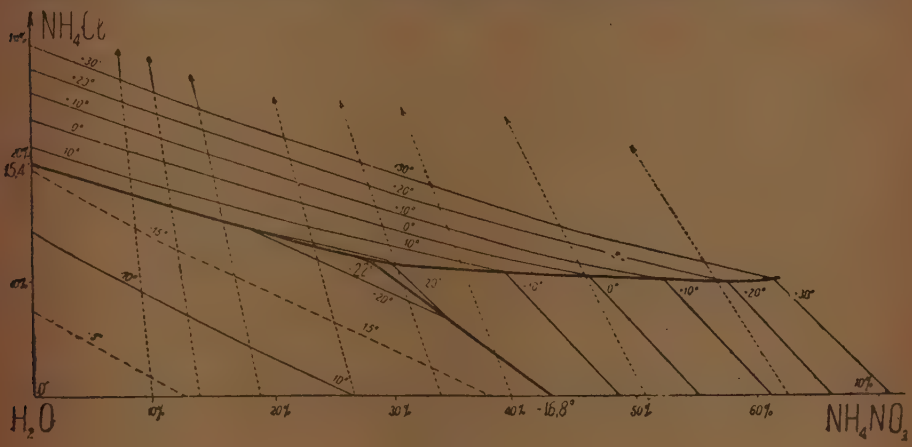

Fig. 1

the solutions of ammonium nitrate in a definite concentration and added to it increasing amounts of ammonium chloride.

For every point of the investigated section the melting temperature of the first crystals that appeared on cooling the solution was determined by the visual method. So was also the temperature at which the last crystals disappeared from the solution on heating. Both temperature estimates differed by no more than 0.2° C.

There was no difficulty in recognizing the separate solid phases: that of ice was typical enough, ammonium chloride was carried down in the form of a silvery dust, and ammonium nitrate in the form of transparent beautiful needle-like crystals.

The salts under investigation were recrystallized. The titre of the ammonium nitrate solution, as established by analysis, was checked by the visual polythermal method with reference to the available literary data on the solubility of this salt. The results concurred perfectly.

% NH_4Cl	% NH_4NO_3	% H_2O	Bottom phase	% NH_4Cl	% NH_4NO_3	% H_2O	Bottom phase
Isotherm —20°C				Isotherm —15°C			
14.4	16.4	69.2	NH_4Cl/ice	18.9	0	81.1	ice
11.2	23.6	61.2	ice	13.8	8.4	77.8	»
7.6	31.6	60.8	»	12.2	12	75.8	»
6.2	34.4	59.4	NH_4NO_3/ice	16.6	10	73.4	»
10.2	30.6	59.2	NH_4NO_3	6	25	69	»
11.2	29.6	59.2	NH_4Cl/NH_4NO_3	2.2	33.4	64.4	»
13	21.2	55.6	NH_4Cl	0	38.6	61.4	»
Isotherm —10°C				Isotherm —5°C			
13.8	0	86.2	ice	7.5	0	92.5	ice
8.2	9.2	82.4	»	1.6	9.9	88.5	»
6.6	13	80.4	»	0	13	87	NH_4NO_3
4	16.2	79.8	»				
0	26.2	73.8	»				
0	48.5	51.5	NH_4NO_3				
10.4	48.8	40.8	NH_4NO_3/NH_4Cl				
11.4	35.6	53	NH_4Cl				
12.8	29.8	57.6	»				
14.2	22.8	63	»				
16.2	15.8	68	»				
17	11.4	71.6	»				
18.4	8	73.6	»				
19	0	81	»				
Isotherm 0°C				Isotherm +10°C			
23	0	77	NH_4Cl	25.2	0	74.8	NH_4Cl
20.4	8	71.6	»	23.6	7.8	68.6	»
19.4	11.2	70.4	»	21.4	10.8	67.8	»
18.2	15.4	66.4	»	18.2	15.4	66.4	»
16.2	22.2	61.6	»	16	22.4	61.6	»
14	29.4	56.6	»	14.2	29.4	56.4	»
12.6	35	52.4	»	12.6	35	52.4	»
10.4	38.4	41.6	NH_4Cl/NH_4NO_3	11	45	44	»
Isotherm +20°C				Isotherm +30°C			
27.5	0	72.5	NH_4Cl	29.6	0	70.4	NH_4Cl
24.5	7.8	67.7	»	26.4	7.4	66.2	»
23.8	10.6	65.6	»	25.2	10.4	64.4	»
22.4	14.4	63.2	»	23.8	14	62.2	»
20	21.2	58.8	»	21.6	20.5	57.9	»
19.4	28	52.6	»	19.4	27.4	53.2	»
15.8	33.8	50.4	»	17.6	31.2	51.2	»
13	44	43	»	14.2	41.4	44.4	»
9.8	56	34.2	»	11	55.4	36	»

As a contribution to the study of this system literary data on ammonium chloride—water and ammonium nitrate—water binary systems were subjected to verification, and nine cross-sections directed to ammonium chloride of 10, 13.8, 19, 26.7, 34, 39, 40, 50.7 and 62.2% concentration were made.

From these sections polythermal data were taken across 5° C for the field of ice and across 10° C for the fields of both salts. The isothermal data are tabulated and represented in Fig. 1.

For a more precise location of isothermal points on the equilibrium curves of two crystalline phases (ice—NH_4Cl, ice—NH_4NO_3 and NH_4Cl—NH_4NO_3) a projection of these curves (crystallization tree) upon the H_2O—NH_4NO_3 side was constructed (Fig. 2). This projection makes it possible more precisely to establish the position of the triple point, whose temperature was determined in a special experiment.

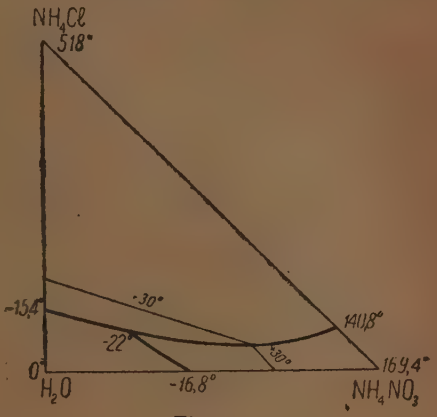

Fig. 2

Conclusions. The polytherm of the system ammonium chloride—ammonium nitrate—water has been explored at full.

From the polytherm diagram it may be inferred that the salting-out action of ammonium nitrate increases with temperature. This is made clear by the fact that the curve of joint solubility approaches the lateral side water—ammonium nitrate. Comparing the cross-sections, one finds that the temperature coefficient of the solubility of ammonium chloride grows with the concentration of ammonium nitrate.

The triple point lies at—22° C. It corresponds to 27% NH_4NO_3+11.4% NH_4Cl+61.1% water. In Fig. 2 is presented a polytherm based on the data of this investigation and on literary data ([1]), including those available for the binary system NH_4NO_3—NH_4Cl. It may be seen that above 30° C the salting-out action of ammonium nitrate goes down and the joint solubility curve runs again away from the lateral side H_2O—NH_4NO_3.

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PHYSICAL CHEMISTRY

ON THE FUNDAMENTAL EQUATION OF THE DYNAMICS OF PHASE TRANSFORMATIONS

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It is well known that the Clausius-Klapeyron equation is the fundamental equation of the *statics* of phase transformations, as it gives the relation between pressure and temperature for two phases *in equilibrium*, when the velocity of the transformation $\left(\frac{d\alpha}{dt}\right)$ is zero.

In the present communication it will be shown that in the general case, when $\frac{d\alpha}{dt} \neq 0$, where α is the relative mass of the second (index 2) phase, we have the following fundamental equation of the *dynamics* of phase transformations:

$$\rho \frac{d\alpha}{dt} = Q \frac{\delta T}{T_0} + (v_1 - v_2)\,\delta p, \tag{1}$$

where Q is the heat evolved in the conversion of the first phase into the second one (when $\delta p = 0$;) v_i, the specific volume of the i-th phase; $\delta T = T_0 - T$, the difference between the temperature at the equilibrium and the temperature of the examined system of two phases T; $\delta p = p - p_0$, the difference between the external pressure and the pressure at which T_0 has been determined; ρ, the coefficient of internal resistance, which decreases with increasing temperature, according to the equation:

$$\rho = \rho_0 e^{\frac{E}{RT}}. \tag{2}$$

From the fundamental equation of dynamics (1) we obtain as a particular case: 1) when $\frac{d\alpha}{dt} = 0$, the Clausius-Klapeyron equation

$$\frac{\delta T}{\delta p} = \frac{T_0}{Q}(v_2 - v_1); \tag{3}$$

2) if instead of $\frac{d\alpha}{dt} = 0$ in equation (1) we assume $\delta p = 0$, we shall obtain the equation found formerly for the rate of conversion as a function of temperature (obtained through analysis of the mechanism of the process on the interface of two phases at their transformation [1]).

For the calculation of the density of the flow j of any physical value (near the equilibrium of the thermodynamic system), additionally to the second principle we shall introduce here the following general relation:

$$j = \lambda \operatorname{grad} X, \tag{4}$$

where X is any function that determines equilibrium and satisfies the condition $\frac{dX}{dx_k} \to 0$ when $j_k \to 0$ $(k, k' = 1, 2, 3)$. Thus, expanding the components j into a power

series of X and taking only the first terms of the expansion (near the equilibrium), we obtain the equation (4), where λ in a general case is a tensor which becomes, however, a scalar in the case of isotropy, when the direction of j coincides with that of grad X.

In a particular case, when we have two phases in contact, it follows from (4) that

$$j = \lambda_{12}(X_1 - X_2). \qquad (5)$$

This can, after all, be obtained directly, as the first term of the expansion, seeing that near the equilibrium $X_1 \to X_2$.

One can easily see that *the laws of electroconductivity, thermoconductivity and diffusion, as well as the expression for the flow of phase transformation* $\left(j = \frac{d\alpha}{dt}\right)$ *derived by the author for the case when* $p=0$ *and when* X *has thus the meaning of the free energy* F [1], *are particular cases of* (4) *and* (5). In the case of phase transformation discussed here, when the external pressure $p \neq 0$, we must for X take instead of F the Gibbs potential (referred to unit mass)

$$g_i = F_i + pv_i \quad (i = 1, 2). \qquad (6)$$

Now the equation (5) may be rewritten as follows*

$$\rho \frac{d\alpha}{dt} = g_1 - g_2, \qquad (7)$$

where

$$\rho = \frac{1}{\lambda_{12}}.$$

Substituting the value of g_i from (6) and taking into consideration the familiar expression for the difference between the free energies of two phases([2])

$$F_1 - F_2 = Q\frac{T_0 - T}{T_0}, \qquad (8)$$

we immediately arrive at (1). As for the variation of ρ with temperature, this dependence has formerly been deduced directly from ([1]).

Now let us consider some applications of equation (1) to the solution of basic problems of the theory of phase transformations.

1. It is known that maximum work is obtained from a system having an excess of free energy when the process is quasi-stationary and at the limit $\frac{d\alpha}{dt} = 0$. In a particular case, when we have a supercooled phase (for instance, water) in a cylinder with a piston, we can apply to the piston a pressure p_{cr}, satisfying equation (3) at a given degree of supercooling, and thus achieve equilibrium. For a given degree of supercooling this pressure is the limiting pressure at which water can still be converted into ice spontaneously (at $T = \text{const}$) (the outlet of heat and the condition $T = \text{const}$ is secured by means of a corresponding thermostat). At this maximum pressure p_{st} we naturally shall have maximum work. *Power, however, in the case of this quasi-stationary process equals zero*, because when $p \to p_s$, we have $\frac{d\alpha}{dt} = 0$ at the limit, the right-hand side of (1) becoming zero for $p = p_{st}$.

The question arises, at what pressure p_m will then the system work with maximum power. In order to answer it, we must find the maximum of the product $\frac{d\alpha}{dt}p$. Sustituting in the equation $\frac{d}{dp}\left(\frac{d\alpha}{dt}p\right) = 0$ the value of $\frac{d\alpha}{dt}$ from equation (1) we obtain

$$p_m = \frac{1}{2}p_{st}. \qquad (9)$$

Thus, *maximum power is obtained at a pressure two times as low as the equilibrium pressure.*

* Equation (7) can as well be obtained by analysing the conversion mechanism (cf.) ([1]).

2. It is known that if a supercooled phase (or, in general, a phase not in equilibrium) is left alone, i. e. if the possibility of a «seed» is removed, its conversion into a stable phase takes place with an acceleration (the phenomenon of «induction», when in the initial part of the kinetic curve we have $\frac{d^2\alpha}{dt^2}>0$). Owing to this, we can conventionally introduce the time of retardation of the conversion τ. This symbol is understood to mean the time necessary for the conversion of the initial phase into a certain quantity of a new one. This quantity α_0 is small and considered as a parameter constant for a given family of curves (in case the process has a jump in velocity, as for instance in autogenetic reactions, the time of retardation has no more this conventional character).

The experiment shows (in agreement with the theory) that the kinetic curves of phase transformations satisfy, in first approximation, the rule of similarity, i. e. they can be made to coincide by changing the scale of time. Selecting thus as a unit of time the time of retardation τ, we shall have an equation of the type

$$\alpha = F\left(\frac{t}{\tau}\right), \tag{10}$$

where F is a function universal for a given family of curves. Owing to the existence of the rule of similarity, *the determination of the time of retardation as a function of p and T is a very important problem of the theory of phase transformations. The solution of this problem gives the possibility, using one kinetic curve for a given p* and T*, to draw curves for any p and T.*

Let us give the solution of this problem on the basis of (1) and (10). From (10) after differentiating and subsequent exclusion of $\frac{t}{\tau}$ we have

$$\tau \frac{d\alpha}{dt} = \Phi(\alpha), \tag{11}$$

where Φ is also a certain universal function of α. Comparing (1) and (11) and taking into consideration (2), we obtain for the time of retardation of phase transformations, as a function of pressure and temperature, the following expression:

$$\tau = \frac{\tau_0}{Q\delta T + T_0(v_1 - v_2)\delta p} e^{\frac{E}{RT}}, \tag{12}$$

where τ_0 is a constant, dependent on neither p nor T, but solely on the selection of α_0.

Let us note that a particular use of formula (12) is in explaining the possibility of the so-called *glassy*, as well as of the *amorphous* state. Indeed, in the case of sufficiently rapid and strong supercooling, carried out so that the phase not in equilibrium has no time during the cooling process to pass into a stable phase, we, if the final temperature T is low enough, can obtain at the expense of the coefficient $e^{\frac{E}{RT}}$ as great a time of retardation and, accordingly, as small a rate of conversion, as we wish. Thus, *we obtain a quasi-stable condition of the unstable phase and, consequently, an extremely slow process of «ageing», i. e. of conversion into the stable phase.*

But a by far more important application of the formula (12) is to the construction of a quantitative theory of steel quenching, in particular of the theory of quenching under pressure. If, in fact, $\delta p=0$, (12) passes into an expression found formerly by the author for the time of retardation as a function of temperature ([1]). This equation affords the possibility to build a basic diagram of the steel quenching and tempering processes, whereby the diagram of Davenport and Bein is meant. With this purpose in mind, a family of curves $t = a\tau(T)$ is plotted on the coordinatenet T, t, where τ is determined by means of (13) and a is a parameter taking different values for different curves. *The general expression found for the time of retardation (12) gives the possibility of finding the curves of the diagram of Davenport and Bein and their shifting effect in the presence of pressure, whereby a foundation is created for the theory of steel quenching and tempering to be built upon.*

3. The physical sense of the Davenport and Bein diagram [the method for its theoretic construction on the basis of (12) has been given in p. 2] consists in that

it brings out the time of conversion up to a given α at different T. It is considered in this case that process of supercooling in itself, i. e. the transformation from T_0 to T, takes place almost instantaneously, after which T remains constant. When, however, the process of supercooling takes place at a definite velocity, a new very important problem arises—the determination of the time t_0 elapsed until a given degree of conversion α_0 is achieved at a specified non-stationary regime, when the temperature changes in accordance with equation

$$T = T_0 - ut, \tag{13}$$

where u is the velocity of cooling and $-u$ is the corresponding velocity of heating. It should be noted that a similar problem of finding t_0 is very important for chemical dynamics, for instance, in the study of the combustion of gaseous mixtures in internal combustion motors. Therefore it will be shown here that this problem can be solved independently of the shape of function $\varphi(\alpha)$, if only the rule of similarity holds [see (11)]. As for the functional dependence of τ upon t, we have, by virtue of (12) and (13), at $\delta p = 0$ in first approximation

$$\tau = \frac{b_0}{ut} \tag{14}$$

where $b_0 = \frac{\tau_0}{Q} e^{\frac{E}{RT_0}}$.

It may be remarked, moreover, that the method described above is fit to solve the problem for other types of the dependence of τ upon t as well.

Multiplying both sides of (11) by $[\Phi(\alpha)]^{-1}\tau^{-1}\,dt$ and integrating from $t = 0$ to $t_0 = t_{\alpha=\alpha_0}$, we obtain

$$t_0 = \frac{a_0}{\sqrt{u}} \tag{15}$$

where

$$a_0 = \frac{2}{b_0} \int_0^{\alpha_0} \frac{1}{\Phi(\alpha)}\, d\alpha,$$

i. e. is a constant, depending on the parameter α_0.

From (13) and (15) we can find immediately the expression for the hysteresis of phase transformations, i. e. the shift-effect of the transformation point in the case of rapid cooling or heating. Take, indeed, into consideration that when the position of the transformation point is determined from an arrest in the temperature or by some other method of such sensitivity that the beginning of the decomposition can only be noticed, after an amount equal to α_0 has been formed, we shall register the beginning of the transformation, according to (15) and (13), only at the temperature

$$T_0^* = T_0 - a_0\sqrt{u}, \tag{16}$$

this giving the value of the shift of the transformation point $T_0 - T_0^*$. *The equation (16) makes it possible to construct* (for not very high velocities u) *the well known diagram of Wefer and Engel* (the velocity of cooling u is plotted as abscissa and the point of conversion T_0^* as ordinate, this giving the apparent change of the equilibrium diagram).

Thus, it may be seen that the very simple and general equation of the phase transformation dynamics makes it possible to approach the solution of the most important problem in the theory of phase transformations, and to build a quantitative theory of steel quenching and tempering, as well as to explain the phenomena of the glassy and amorphous steel and the «ageing» of phases.

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PHYSICAL CHEMISTRY

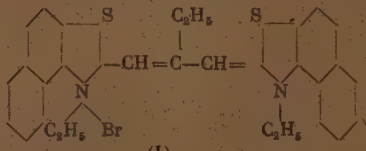
STUDIES ON THE CONCENTRATION EFFECT IN THE CASE OF OPTICAL SENSIBILIZATION

By G. N. GOROKHOVSKY

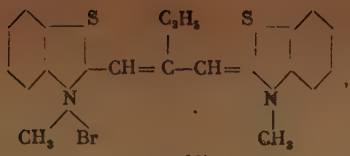
(Communicated by S. I. Wawilow, Member of the Academy, 19.XII.1942)

The additional sensitivity of a photographic emulsion created by the introduction of a dyestuff is known to increase with the concentration of the dyestuff until a maximum is reached, after which it begins to fall. There is qualitative evidence that in a number of cases increase in the concentration of the dyestuff not only causes the additional sensitivity to change in absolute value, but also affects its distribution along the spectrum. Rabinovich [1] has reported a concentrational displacement of the maximum of additional sensitivity in an emulsion sensibilized by erythrosine. Moreover, Leermakers, Carroll and Staud [2] and Davey [3] have observed, with several dyestuffs, the appearance of new sensibilization bands in response to increasing concentration. The phenomenon appears to be of a similar nature with the well known Fogel effect: the reason for the adsorbed dyestuff changing its optical properties with concentration may lie either in the polarizing influence of the electrostatic field of the adsorbent [4, 5], or in the aggregation of the dyestuff molecules upon adsorption. Furthermore the filter effect may also be a factor here; by this I meant the absorption of the dyestuff remaining in the gelatine layer. Inasmuch as the absorption band of the adsorbed dyestuff (the sensibilization band) is always displaced toward the long-wave side of the spectrum with respect to the absorption band of the dissolved dyestuff, the latter may cut off the short-wave branch of the curve of spectral sensitivity.

Positive slides of the Research Institute for Cinematography and Photography and Spektralplatten blau rapid Agfa have been subjected to sensibilization by bathing in freshly prepared solutions of five different dyestuffs. Both kinds of plate gave similar results. None of the selected dyestuffs had the same chemical structure with the others and they differed also with regard to their spectral region of sensibility and their physical-chemical state in the solution. There was an acid dyestuff—the erythrosine, and the other were basic dyestuffs: one carbocyanic dyestuff—the pinacyanol chloride, two thyacarbocyanic of the following structure (sensibilization to yellow-red rays of the spectrum):

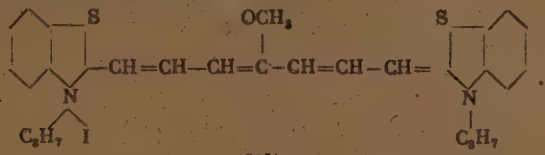


(I)



(II)

and one thyatricarbocyanic (infrachromatic)



(III)

Their concentration varied within a range of 1 to 10 000, the optimal concentration lying somewhere in the middle of this interval ($2—5\times10^{-5}$ moles). The composition of the solvent for the basic dyestuffs was $H_2O : C_2H_5OH = 3 : 2$.

Of the tested dyestuffs the erythrosine appears to give true solutions in a wide interval of concentrations; the basic dyestuffs in water-alcohol mixtures showed a tendency to the formation of colloid solutions. Dye III is remarkably unstable and prone to coagulation. The pinacyanol chloride is liable to polymeric aggregation.

The sensibilized plates were exposed in the spectro-sensitimeter of the State Optical Institute ([6]) and subjected to most careful photomicrometric measurements. After that, curves of spectral sensitivity $\lg S_\lambda = f(\lambda)$ were plotted. The corresponding curve families for three of the investigated dyestuffs (only for the region of added sensitivity) are represented in Figs. 1—3. On these curves the position of sensibilization maxima were determined within 1 mμ. These data are summarized for the five dyestuffs in the table.

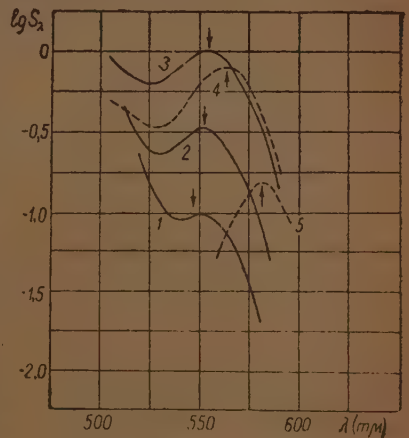


Fig. 1. Sensibilization by erythrosine; concentration of sensibilization bath (in moles): 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2} indicated as *1*, *2*, *3*, *4*, *5* respectively

Furthermore, the gelatine films were coloured by dyestuffs, while bathing fixed plates in the same solutions and in the same way as in the case of sensibilization. By means of a photoelectric spectrophotometer with glass optics absorption curves of these films were taken; the maxima of absorption are given in the table. It was found, especially, that no appreciable colour was taken on by the film, unless the concentration of the sensibilizing bath exceeded the optimal for sensibilization.

From the curves it follows, in the first place, that the high ratio of absorption maxima for the dyes with two absorption bands (all basic dyes) stands in no relation whatever to the high ratio of spectral sensitivity maxima. In many cases strong absorption corresponds to low sensibilizing action, and conversely. In all cases the long-wave band α was the more active photochemically, while the short-wave band β was the less active (different yield of sensibilization?).

In the second place, it appears that when the sensibilizing dyestuff in the bath changes in concentration, a considerable redistribution of its photochemical action along the spectrum takes place. In the region of moderate concentrations

(not above the neighbourhood of 10^{-4} moles) the different dyestuffs have exhibited the following individual characteristics.

Set of tests	Dyestuff	Absorption band	λ_{max} of absorption		λ_{max} of sensibilization					
			Concentration ratio		Relative concentration of dyestuff in bath					
			100	1 000	1	10	100	1 000	3 000—10 000	
			Nos. of curves on Figures 1—3							
			3	4	1	2		3	4	5
A	Erythrosine	—	540	540	550	552	Optimal concentration	554	564	582
P	Pinacyanol	α	622	—	631	630		629	630	667
		β	567	567	583	583		583	581	
E	Dyestuff III	α		735		757			756	
		β		575		584			584	
F	Dyestuff I	α	596	—	596	600		605	622	
		β	547	548	—	—		556		
C	Dyestuff II	α	552	548	567	565		[575]	602	605
		β	—	518	—	—				

1. Erythrosine with increasing concentrations shows a slight shift of the sensibilization maximum, but the curve of spectral sensitivity scarcely changes its shape.

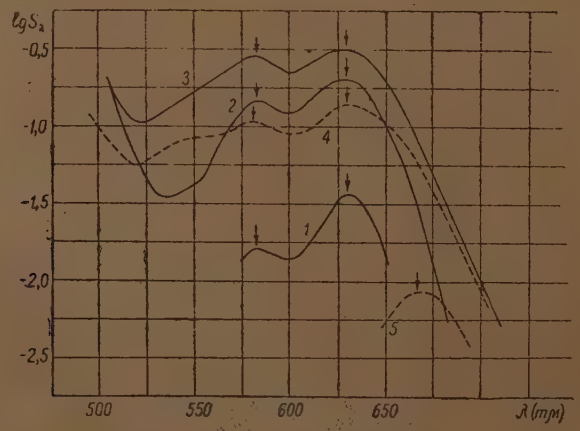


Fig. 2. Sensibilization by pinacyanol; concentration of sensibilization bath (in moles): 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 4×10^{-3} indicated as *1*, *2*, *3*, *4*, *5*, respectively

2. In the case of pinacyanol and dye III the spectral distribution of sensitivity and, accordingly, the position of the maximum in either of the sensibilization bands (α and β) is practically invariable with concentration. Yet, with increasing concentration of the dyestuff the growth of the absolute value of sensiti-

vity is more rapid in the region of the β band, so that the photochemically less active band β overtakes and even outstrips the more active band α.

3. In the case of dyes I and II the increase in the concentration of the dyestuff widens the sensibilization band; the maximum becomes more gentle and is somewhat shifted along the spectrum. What appears to be a probable explanation of the widening of the sensibilization band in the short-wave region, is that the low-active band β situated close to the α band begins to exert its influence. Yet, the widening is also observed in the long-wave region where these dyestuffs have no specific bands of absorption. Hence we may speak of the α band widening proper.

As for high concentrations of the dyestuff (above 10^{-4} moles), when the gelatine layer takes colour rather richly, there is observed the same picture in all cases—the short-wave branch of the spectral sensitivity curve is depressed with

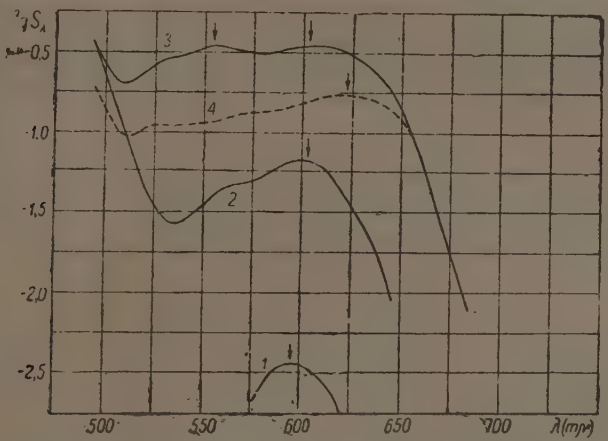


Fig. 3. Sensibilization by dye I; concentration of sensibilization bath (in moles): 7×10^{-7}, 7×10^{-6}, 7×10^{-5}, 7×10^{-4}, denoted by *1*, *2*, *3*, *4*, respectively

the result that a considerable displacement of the maximum toward the long-wave side takes place. At the same time the whole curve $\lg S_\lambda = f(\lambda)$ begins to sink, i. e. a general desensibilization of the emulsion takes place. If, in fact, the absorption by the gelatine layer D_λ is allowed for in determining the spectral sensitivity of the emulsion containing a high concentration of the dyestuff ($\lg S_\lambda^* = \lg S_\lambda + KD_\lambda$), then the resultant curve $\lg S_\lambda^* = f(\lambda)$ will be quite alike with the spectral sensitivity curve for an optimally sensibilized emulsion.

To my opinion, the cause of the stated differences in the concentrational effect on sensibilization should be thought in the different physical-chemical conditions of the various dyestuffs in solution. The molecularly dispersed erythrosine, as well as pinacyanol and dye III, which, as dyes with distinctly expressed colloidal structure, are on the other end of the scale, appear to present molecular complexes of such completeness that adsorption, irrespective of concentration, fails to produce any change in the degree of aggregation of these dyestuffs and, accordingly, in their optical properties beyond that corresponding to the effect of Fogel. Between these the thyacarbocyanic dyes I and II are intermediate. They seem to be in such a state of aggregation when adsorption is apt to call forth an active interaction between the still rather labile molecular complexes. The result is that new structure units are formed with a wider spectrum of «collectively owned» electrons. Under these conditions the absorption band of the absorbed dyestuff can vary in width, according to the initial concentration of the dyestuff, i. e. the initial degree of its dispersion.

Of course, these are merely conjectures which to become a certainty must be proved by comparing spectral sensitimetric data for a great number of different

dyestuffs with data on the state of dyestuffs, while in the dissolved or adsorbed condition, obtained by different physical-chemical methods.

With concentrations 5 to 10 times the optimal one or even higher, the filter effect comes to the fore, that is the absorption of the dyestuff present in the gelatine. This effect is accompanied by a general desensibilization of the emulsion and entirely overshadows those subtler changes in the photochemical properties of the dyestuff which have been mentioned above and are likely to take place in this interval of concentration also.

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BIOCHEMISTRY

THE ANTIHAEMORRHAGIC EFFECT OF VITAMIN K_3

By V. V. BABUK

(Communicated by A. N. Bach, Member of the Academy, 17. I. 1943)

It has been proved by a number of authors that the vitamins K_1, K_2, as well as methyl-naphthaquinone act only in those cases when there is hypoprothrombinaemia. When the blood shows a normal coagulation, they prove absolutely inefficacious, producing no appreciable effect upon haemorrhages attending haemophilia, thrombopaenia, essential haematuria and other affections, proceeding with a normal content of prothrombin in the blood plasma.

The antihaemorrhagic substance, isolated by D. M. Mikhlin from stigmata of Indian corn and given by the author the name of vitamin K_3, favourably compares in this respect with the rest of remedies.

When introducing it in the blood of dogs and rabbits, Mikhlin obtained a considerable increase in the rate of blood coagulation (twice, thrice and even more). The test-animals were subjected to no experimental avitaminosis and showed prior to the experiment normal blood coagulation. The latter circumstance being of exceptional practical importance, we addressed ourselves to D. M. Mikhlin, begging him to put his preparation at our disposal for clinical checking.

Starting our investigation, we set before ourselves the following problems:

1) To establish the dose and the length of the effect of vitamin K_3 administered *per os* to patients, showing normal blood coagulation.

2) To determine the effect of vitamin K_3 when introduced subcutaneously and intramuscularly.

3) To clear the mechanism of the action of vitamin K_3.

The effect of vitamin K_3, administered *per os* was tested by us on 59 patients, suffering from various diseases. The most efficient dose proved to be 40—50—100 mg of the raw matter, extracted with alcohol (4—10 cm^3) from a corresponding amount of dry maize stigmata.

In the case of normal blood coagulation, vitamin K_3 has been found efficient when given *per os* in the form of alcohol extract. The effect sets in after an hour and reaches its maximum 2 hours after the drug has been taken. Within the subsequent 4—5 hours the effect ceases gradually. It should be noted that the effect of vitamin K_3 is attended by no side phenomena; it acts without failure, increasing the rate of blood coagulation. However, it proves most efficacious in patients whose blood coagulation is reduced. This should be taken to mean that in diseases accompanied by a reduced blood coagulation the use of vitamin K_3 will be most efficient. Taking into account the period of activity of the drug, it must be prescribed every 4 hours.

In 20 cases we administered vitamin K_3 *per os*, but in the form of oil solutions, prepared with cotton or apricot oil. The dose was the same. As it should be anticipated, the effect of oil solutions sets in later, though it lasts for the same period of time. In most of the cases the rate of blood coagulation shows an increase 5—7 or

even 8 hours after the drug was taken. There were, however, occasions when the effect set in earlier (after 2—3 hours). The resorption of the vitamin is evidently affected by the condition of the intestine, by liver and bile-duct disorders, by the character of food and other causes. It should, however, be pointed out that whenever an active preparation was used, the effect was invariably a positive one.

As control served 10 patients who received no vitamin at all; when tested for coagulation under similar regime and at the same hours of the day their blood showed no appreciable variations.

Since most of the patients were given vitamin K_3 in the form of alcohol extract, it seemed of interest to clear the effect of alcohol upon the coagulation of blood. With this aim in view ten patients were given 10 cm^3 of alcohol instead of the same amount of vitamin extract. In 5 of them the blood showed a reduced rate of coagulation, in 4 there was no effect at all, and in one case only there was the same increase in the rate of blood coagulation. Hence it is quite evident that the increase in the rate of blood coagulation is by no means due to alcohol, but to the vitamin alone.

In a number of diseases the internal use of the vitamin may not only be inefficient, but even impracticable; this is the case, e. g., during the first days upon surgical interventions applied to the stomach. We tested therefore the effect of vitamin K_3 on 17 patients to whom the vitamin was given either subcutaneously, or intramuscularly. In both cases apricot oil solution was used. Subcutaneous injections of the vitamin K_3 are likewise accompanied by enhanced blood coagulation; the effect sets in 3—4 hours upon injection and lasts for the same period of time. Quite unexpected and rather striking results have been obtained in the case of intramuscular injections. The rate of coagulation is rapidly increased (after 1—3 hours) and remains stable during a long period of time, from 7 to 10 hours. The only drawback of subcutaneous and intramuscular injections of the preparation is that they are painful.

Vitamin K_3 has been introduced by us twice intravenously with an excellent effect. The dose was 90 mg of vitamin in 9 cm^3 of alcohol. Coagulation after Sitkowsky-Egorov in seconds previous to the injection was 195; after 30 min.—130; after 1 h. 30 min.—125; after 2 h. 30 min.—120; after 3 h. 30 min.—135; after 4 h. 30 min.—135 and after 5 h. 30 min.—130.

In spite of the fact that we could notice no side phenomena, accompanying intravenous injections, and the effect of the vitamin was both rapid and stable, we refrained from continuing the experiments, since intravenous injections of the vitamin in 96—98° alcohol may involve some risk.

The data referred to above are sufficient to insist upon a wide-scale application of vitamin K_3 in the most diverse cases; in haemorrhages, attending cholaemia, in parenchymal haemorrhages, following traumas, in pulmonary haemorrhages due to wounds of the thorax or to lung tuberculosis, or haemorrhages from granulations after wounds, in haemorrhoids, stomachal or intestinal haemorrhages of ulcerous origin, as well as in the case of haemorrhagic diathesis, epistaxis, some gynecological haemorrhages, and in many other cases when surgical haemostatic procedures are counterindicated.

Vitamin K_3 should also be applied for the sake of prophylaxis before operating such cases as are accompanied by hyperprothrombinaemia (mechanical jaundice, biliary fistula) and previous to operations followed by various degrees of haemorrhages (operations of the gall-bladder and bile ducts, stomach, prostata etc.).

The mechanism of the action of K-group vitamins has not been sufficiently well investigated as yet. By injuring the liver of their test-animals with chloroform, or by partly removing it, Smith, Warner and Brinkhaus have shown the important rôle played by the liver in the synthesis of prothrombin. The same authors have established that the tendency to bleeding observed in persons suffering from obturation jaundice is due to prothrombin deficiency. They have also demonstrated that when there are given, along with salts of the bile, extracts of *Lucerna*, rich in K vitamins, the content of prothrombin in the blood plasma of those patients increases rapidly.

Warner has shown that the removal of a part of the liver results in a reduced concentration of prothrombin in the blood plasma of rats. Snell and Butt point out that certain liver disorders (cirrhosis, atrophia, chronic hepatitis), as well as injuries of the liver, are not unfrequently attended with hyperprothrombinaemia.

In order to test vitamin K_3 for its ability to increase the concentration of prothrombin in the blood plasma, we examined the prothrombin in 10 patients prior to and after they had been given vitamin K_3; the concentration of prothrombin was determined according to Smith during the vitamin's action. We have been able to show that the effect of vitamin K_3 is to increase the concentration of prothrombin in the blood plasma.

It may consequently be assumed that the increase in the rate of blood coagulation is accounted for by the stimulating action of vitamin K_3 and of other vitamins of the same group upon the synthesis of prothrombin in the liver. It should also be pointed out that the stimulating effect of maize stigmata upon the biliferous function of the liver had already been shown by Allaknhverdibekov.

Aside from this, we also examined the blood of 18 patients for thrombocytes before they had been given the vitamin K_3, and during the effect of the latter, under various modes of application.

The content of thrombocytes showed but insignificant fluctuations; we feel therefore not justified to associate the increase in the rate of blood coagulation observed in the case of vitamin K_3-treatment with the variations in the number of thrombocytes.

In concluding I wish to emphasize the necessity to urge the production of vitamin K_3 and the supply with it of medical installations, clinics and, in the first line, of military hospitals.

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GENETICS

SYNTHESIS OF WINTER-HARDY WHEATS

By M. I. SALTYKOVSKY and E. S. SAPRYGUINA

(Communicated by A. A. Richter, Member of the Academy, 26. III. 1943)

The great economic importance of winter-hardy varieties of wheat urged the breeder to develop his work on winter wheats in this direction. But though conducted on a large scale, breeding for wheat winter hardiness goes ahead very slowly. One of the bottlenecks that check its progress is the great scarcity of winter-hardy forms among the world's wealth of wheat varieties. Hybridization, which is the most widespread method in wheat breeding, has but seldom yielded transgressive forms in respect to winter hardiness, and even when such forms were obtained, their resistance did not greatly exceed that of the more winter-hardy parent. Exceptions to this rule are very rare. This state of affairs continues, because the selection of pairs for the crossing has never been put on a theoretical basis.

We were able to show in our previous communications that wheats in the phase of tillering, in which state they pass winter if sown in autumn, have their resistance to cold functionally correlated with the length of two developmental phases: the phase of vernalization and the light phase (1). This discovery gave rise to a cycle of thorough experiments on the hereditary transmission of the winter habit and winter hardiness in wheat and wheat-quitch hybrids. Some data on the transgressive variation of cold-resistance for a single combination has been published in (2). In the present paper part of the materials are presented obtained by an extensive verification of the significance of the two above-mentioned developmental phases in the inheritance of winter hardiness. The object of that work was to reveal the magnitude of the transgressions of winter hardiness for the hybrids of spring variety—winter variety combinations.

Material and Method of Investigation. F_1 were grown in 1936 after spring planting. In the autumn of the same year F_2 were sown in the field in four replications, 250 grains in every replicate of each of the combinations. Two of the replicates passed the winter under a deep snow cover artificially created. The other two under natural snow cover. These were severely affected by the cold. All of the latter plants were taken individually to test the older generations of their progeny for winter hardiness. In the year 1937/1938 the hybrid lines of one replicate were tested under field conditions at Bezenchuk. The field experiments with F_3 were conducted in similar surroundings. Aside from field experiments, laboratory tests for cold-resistance of the hybrid lines were run with another replicate of F_3, involving the two subsequent generations. All the hybrid lines of F_3 passed winter better than the more resistant of the parents. Part of the lines approaching in the indices for winter hardiness to the resistant parent was, moreover, used as material for the study of the following generations. Within those combinations from which no transgressive forms had been obtained among F_3, all the most resistant lines were taken for testing the older generations. This was done in order to check our statement concerning the selection of pairs in cross-

ing for winter hardiness. All work down to the fourth generation was carried out at Bezenchuk (province of Kuibyshev), and the F_4 were experimented upon at Dnepropetrovsk. Here the transgressive hybrid forms in F_4 and F_5 were subjected to a more detailed investigation, according to our differential field method for studying the winter hardiness of winter cereals (3). The sowing of the hybrids F_4 in 1938/39 and of F_5 in 1940/41 was done in duplicate on long one-row plots. To one side of every hybrid line the winter-hardy parent was planted. On the other side stood the winter variety *Hostianum* 237. Among the widespread varieties of the steppe part of Ukraine this one is the most remarkable for winter hardiness. In winter three types of conditions were created in the experimental field. There was a strip of ground on which snow was removed two times during the winter in order to chill the plants. On another strip the snow was allowed to accumulate throughout the winter. The soil under the deep snow cover was never frozen during 1938/39. These were very suitable conditions for trying the power of the hybrids to withstand the rotting influence of moisture. On the third strip the plants went through winter under natural snow cover. All the latter plants passed winter very well in the two years of the experiment (F_4 and F_5). Therefore the data relating to them are here not referred to in the tables. In the experiment of 1940/41 apart from the winter varieties Kooperatorka and *Hostianum* 237 one more standard variety of Ukrainian winter-wheat, the Ukrainka, was planted for comparison with the hybrids. With regard to winter hardiness, Ukrainka is intermediate between Kooperatorka and *Hostianum* 237. No rotting conditions could be created during the winter of that year. For that reason the data on the fifth generation are of interest only as far as the conditions of freezing are concerned.

The hybrid material in our investigation consisted of 15 combinations of spring and winter varieties of wheats crossed in 1935. In all of these combinations the parental pairs were selected to represent different lengths of developmental phases. Of the fifteen combinations only five yielded transgressive forms. All of these five combinations satisfy the principle mentioned at the beginning of this paper, relative to the selection of pairs for the synthesis of winter-hardy forms. Namely, by selecting and crossing wheat varieties as near as possible in cold-resistance, but strongly different in the length of their respective development phases, it is possible to obtain, among the hybrids of older generations, more cold-resistant forms than their parents. Of the remaining 10 combinations, within which, according to our hypothesis, there could be no transgression of winter hardiness, and which were taken as controls, no one produced transgressive forms, although the conditions of experiment were equal for all of the 15 combinations. This fact is no less convincing a demonstration of the correctness of the above principle than the repeated appearance of transgressions within a single combination and in a series of new combinations. The question arises, How great is the transgression in the crosses between spring and winter wheats? In Table 1 the average data on winter hardiness (cold-resistance and resistance to the rotting influence of moisture) are given for 3 years and for 3 combinations; the surplus of the hybrid plants that withstood winter over the winter-hardy parent plants and over those of the standard varieties is given in per cent. The data obtained show a very diversified extent of transgressive variation for the hybrid lines tested. Some lines exhibit a small increase of winter hardiness compared to the resistant parent. Other hybrids give a higher surplus of survived plants under conditions favourable for moisture rotting than under freezing. There also were hybrids which under any conditions of hibernation and in all the 3 years of experiment showed considerable resistance. Such were the lines 117/29, 117/65a, 117/64, 178/7, 169 30. In these lines the surplus of plants that outlived winter, compared to the resistant parent, is very considerable. What is more, all these hybrids are more winter-hardy than Ukrainka. Some of the hybrids approach in winter hardiness to *Hostianum* 237. The line 117/64 after two years of comparative testing with *Hostianum* 237 showed a somewhat stronger winter hardiness.

The laboratory method of investigating cold-resistance in the combinations *Lutescens* 4952 × Kooperatorka, *Lutescens* 3483 × Kooperatorka, *Milturum* 321 ×

Kooperatorka gave similar results to those obtained in the field experiments. In the laboratory tests, besides the mentioned combinations, there were two

Table 1

Winter Hardiness of Wheat Hybrids

Combinations	Nos. of hybrid lines	Surplus of survived plants under freezing over the varieties						moisture rotting over the varieties	
		Kooperatorka			Ukrainka	*Hostianum* 237		Kooperatorka	*Hostianum* 237
		F_3	F_4	F_5	F_5	F_4	F_5	F_4	F_4
Lutescens 4952♀ × Kooperatorka♂	117/39	16	27	30	−6	−15	−31	21	−5
Do	117/43	17	3	—	—	−32	—	16	−19
»	117/64	34	39	71	35	4	10	33	0
»	117/656	23	12	42	6	−19	−19	39	−5
»	117/65a	25	16	50	14	−14	−11	17	−9
»	117/12	5	13	45	6	−7	−18	10	0
»	117/29	20	24	53	14	−7	−10	10	2
»	117/30	3	0	—	—	−25	—	13	−6
Lutescens 3483♀ × Kooperatorka♂	178/7	13	18	53	14	−18	−10	30	−10
Do	178/9	22	10	40	1	−12	−23	27	−2
Milturum 321♀ × Kooperatorka♂	169/29	12	13	—	—	−17	—	5	−20
Do	169/30	27	20	58	19	−8	−5	26	−8
»	169/33	33	9	—	—	−19	—	8	−14

more which gave strong transgressions (because of the lack of grain no field experiments were run with these combinations). In Table 2 we give the experimental

Table 2

Cold-Resistance of Transgressive Forms of Wheat Hybrids in F_3 and F_4

Material	Nos. of lines	Per cent of survived plants after chilling						
		F_3				F_4		
		−17°	−19°	−22°	−23°	−15°	−17°	−19°
Lutescens 3483		0	0	0	0	0	0	0
Lutescens 3483 ♀ × *Erythrospermum* 534♂	175/1	90	85	85	75	98	67	22
Do	175/2	100	90	84	67	86	79	35
»	175/5	95	85	67	60	68	26	0
Erythrospermum 534		45	40	16	6	22	2	0
Kooperatorka		—	—	—	—	58	38	0
Ukrainka		—	—	—	—	78	62	0
Milturum 321		0	0	0	0	0	0	0
Milturum 321 ♀ × *Erythrospermum* 534♂	167/3	—	—	—	—	57	27	0
Do	167/7	—	—	—	—	33	29	0
»	167/8	79	63	69	26	97	70	40

material obtained by testing the two additional combinations in the laboratory —*Lutescens* 3483 × *Erythrospermum* 534, *Milturum* 321 × *Erythrospermum* 534. The transgressive variation, as shown in Table 2, is by no means less and is even greater than in the case of the 3 combinations above-mentioned.

To give a clear idea as to how great is the transgression of cold-resistance in the hybrids from these crosses as compared to the winter-hardy parent, one should say some words about the cold-resistance of the latter. *Erytrospermum* 534 has a very poor resistance to cold. It is as much inferior in this respect to the variety Kooperatorka, as the latter is to *Hostianum* 237. The hybrids between *Erythrospermum* 534 and two varieties of spring-wheat yield forms not merely more resistant than the two parents and Kooperatorka, but even more resistant than Ukrainka. Thus, we here have a jump of cold-resistance over several classes of winter-wheat varieties. Apart from the described transgressions of winter hardiness in the five crosses of 1935, transgressive forms were obtained also from several more combinations in the subsequent years, when the crossing was carried out according to our principle of pair selection. These experiments have not been carried as far as to the older generations.

The magnitude of transgression in wheat hybrids can be brought out more clearly still by the following comparison. For a very great number of to-day varieties of winter-wheats of Europe, Asia and North America the variation of winter hardiness lies between the resistances of the winter varieties *Erythrospermum* 534 and *Hostianum* 237. The hybrids produced in the experiments, in spite of the fact that a very limited initial material was at our disposal (only 250 grains of F_2 in each combination) and the selection made only once, showed the same amplitude of variation. The comparison brings us to the question as to the evolution of winter-hardy winter-wheats. In southern regions (south-west of Asia, South Europe) two basic biological types of wheat are used in winter sowing. The one type are winter-wheats with very low cold-resistance. They have a rather long phase of vernalization, and a short or moderate light-phase. The second type are spring-wheats with a short phase of vernalization and a long light-phase; these are late to ripen if sown in spring. The crosses between the two biotypes mentioned give large and frequent transgressions of winter hardiness. Being sown in a single population, these biotypes had good opportunity for spontaneous crossing, because this phenomenon is rather usual with wheats. The spontaneous hybrids between winter-wheats of poor winter hardiness and late spring-wheats present one of the main initial ways followed by the evolution of winter-hardy spring-wheats.

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PALEONTOLOGY

GLOSSELYTRODEA FROM THE JURASSIC SHALES OF THE COAL BED SOGJUTA

By O. A. MARTYNOVA

(Communicated by A. A. Borissiak, Member of the Academy, 3. I. 1943)

In summer 1942 I was working at the Sogjuta coal deposit at the head of a party, sent by the Paleontological Institute of the Academy of Sciences of the

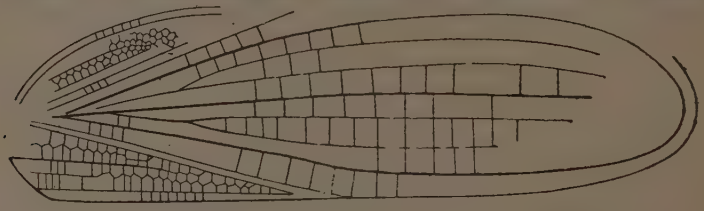

Fig. 1. *Mesojurina sogjutensis* g., sp. n., No. 371/343

USSR. The object of this expedition was to investigate the fossil fauna of Mesozoic insects, discovered in 1928 by N. A. Kulik.

The coal deposit of Sogjuta outcrops at a length of 12 km along the southern coast of the lake Issyk-kul. Numerous ravines, descending to the lake, break the Jurassic beds and form beautiful outcrops along its banks. Especially advantageous in this respect is the ravine Ak-say which can be divided into the eastern, western and the central. The central Ak-say represents an even surface, covered with alluvia of granitic pebble and gravel; along its banks there are outcrops of Jurassic series, *C* and *H*. The latter is insect-bearing, and has received the name of the «green shale series». Its thickness, as outcropped at the right bank of the central Ak-say, is about 40 m. The series is almost homogeneous with respect to its lithological composition. The fine-grained argillites in the upper part of the bed are slightly arenaceous. The entomological fauna of the green shale series is characterized by a large quantity of remains of *Diptera* and *Coleoptera*. One tenth of the fauna collected is made up of *Mecoptera*. A preliminary determination of these enables one to refer the series to the Lower Lias.

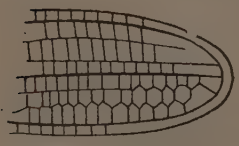

Fig. 2. *Mesojurina sogjutensis* g., sp. n., No. 371/340

Among the numerous imprints of insects (1500 in number), collected in series *H*, there have been found two imprints of elytra of representatives of *Glosselytrodea*. This order has originally been described from deposits of the Kazan stage of the Permian of the Near-Urals (Tikhie Gory at the Kama river), and of the Arkhangelsk department (Soyana river). The representatives of the order *Glosselytrodea* found in the Sogjuta Jurassic belong to a new genus, *Mesojurina* g. n. of the family

Jurinidae M. Zal. A. V. Martynov ([3]) believed *Glosselytrodea* to be an order extinct as far back as during the Permian. However, the finding made at Sogjuta points to the existence of this order during the Mesozoic, and in a slightly altered form with this.

Mesojurina g. n.

Very close to *Jurina* M. Zal. described from the Permian of Tikhie Gory, from which it is distinguished by the veins *MP* and CuA_1 coalescent into a single strong vein, passing along the middle of the wing, and by a finely-cellular field without pronounced veining.

Mesojurina sogjutensis sp. n.

Imprints of elytra Nos. 371/313 and 371/340 (collection of the Paleontological Institute of the Academy of Sciences of the USSR). Both imprints in a good state of preservation. The former represents an almost intact wing: only a part of the costal field and of the distal part of the precostal region are lacking. The second imprint represents the distal half of a wing. The length of the elytrum is 4.7 mm; the width, 1.5 mm.

The shape and mode of veining are similar to those in *J. reducta* Mart. (Permian, Soyana river). The precostal region, somewhat less prominent than in *J. reducta*, is wedging forward over the middle of the wing. A rather clear-cut veinlet runs near the anterior edge parallel to the latter, forming a trim, covered with dense transverse veinlets. Along the posterior edge of the precostal region there runs also a sharp edge-veinlet with a multitude of transverse veinlets. The whole precostal region between the edge-veins is covered with a net of cells; these can scarcely be traced, being extremely fine and strongly deformed by short transverse veinlets. Their arrangement seems to be similar to that in *J. reducta*, except for the ends which are more retorted. Anal region wedge-like. A_1 runs parallel to the posterior edge of the costocubital region. A_2 is a sharp veinlet at the base, it becomes less expressed farther on, and is indiscernible towards the end. Between the anal veinlets and the edge strip there are two feeble veinlets, that run parallel to A_2. The entire anal region is covered with a dense network of transverse veinlets. The costocubital region is, as usually in this order, lingulate, wedging at the base. *SC* and *CuP* encircle the wing and coalesce at its apex, forming a trim. *R* simple, without ramifications as those in *J. reducta*. *MP* coalesce with *R* not far from the base of the wing. *R* and *MP* are slightly bent towards the middle at their ends. *MP* and *CuA* are fused into a single well-expressed and broad vein, which runs straightly along the middle of the wing, unlike that in *J. reducta*. CuA_2 are bifurcate, as in the rest of species of the genus *Jurina*. However, CuA_2 branches in two at the level of coalescence of *MA* and *R*, i. e. near enough to the base of the wing. In the Permian species CuA_2 branches considerably more distally. The segment CuA_{1+2} is equal in length to the segment CuA_2 before the latter branches. Neither of the two branches of CuA_2 reaches the edge of the wing. Their ends are indented, deformed by short transverse veinlets. The wing is covered with a dense network of transverse veinlets like that in *J. reducta*.

The Jurassic representative of the order *Glosselytrodea* differs but little from its Permian analogons, except for the smaller size and lack of a sharp prominence in the middle of the anterior edge of the precostal field, and the coalescence of *MP* and CuA_1 into a single veinlet.

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PARASITOLOGY

DYROMYS NITEDULUS PALL. AS A POSSIBLE RESERVOIR OF THE VIRUS OF TICK RELAPSING FEVER

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In the course of my study of animals serving as a reservoir of the virus of the tick relapsing fever spirochaete, I succeeded to obtain in Leningrad six individuals of *Dyromys nitedulus* Pall. This small animal has been tested for its susceptibility to the infection with the spirochaetae of the tick relapsing fever taken from the collection of strains, collected by the Department of Biology and Parasitology of the S. M. Kirov Medical Academy. The animals were infected by inoculating them infected blood from guinea-pigs infected with spirochaetae of tick relapsing fever of the strain «Kuliab» (from Tadjikistan buildings) transmitted by the ticks *Ornithodorus papillipes*, and of the strain «Tbilissi» (from the station Sanganlug near Tbilissi, turtle-holes) transmitted by *Ornithodorus verrucosus*.

Four experiments have been made with the inoculation of blood from guinea-pigs into the nostrils and the conjunctiva of the eye of *Dyromys nitedulus*. Inocu-

Experiments on Inoculation of *Dyromys nitedulus* Pall.

No. of test animals	Date of inoculation	Mode of inoculation	Strain of spirochaetae	Blood — December 1939													
				7	8	9	10	11	13	14	15	16	17	18	20	21	22
1	4. XII. 1939	Inocul. blood from infected guinea pigs	«Kuliab» and «Tbilissi»	—	+ v. m.	+ m.	+	++	+	++	++	+	+ m.	+ m.	++	++	+ v. m.
2	Do	Do	D	+ v. m.	+	++	~	~	+ v. m.	+ m.	+	+	~	~	+	+ v. m.	+ v. m.
3	»	»	»	+ v. m.	+ m.	++	~	~	+ v. m.	+	++	~	+	++	~	++	~
4	»	»	»	—	—	+ m.	+	++	+ m.	++	~	++	+	+ v. m.	+ v. m.	+ m.	++
5	31. XII.1939	Fed on by infected ticks	«Stalinabad»														
6	Do	Do	Do														

lation was practised by way of simple pricking. The blood for microscopic examination in the smears stained after Giemsa-Romanovsky was taken every day till the spirochaetae disappeared entirely.

All the four animals proved infected, two after three days of incubation, one after four days and one after five days. The blood was at once found to contain great number of spirochaetae. The period during which the latter were detectable under the microscope varied from 21 and 32 days. In two of the animals the spirochaetae stayed persistently throughout this period of time, showing four or five times conspicuous maxima. In *Dyromys nitedulus* No. 1 (see the table) the infection lasted during 32 days; in the course of this period the spirochaetae twice disappeared from the peripheral blood, once for 24 hours, and the other time for four 24-hour-days. One of the animals perished at the climax of the infection, two — on the 5th and 11th day after the disappearance of spirochaetae. The course of the spirochaetosis according to days is shown in the table adjoined.

The experiments described in this note bring out the high sensitivity of *Dyromys nitedulus* Pall. to the intranasal and conjunctival inoculation with the spirochaetae of tick relapsing fever. Now they were to be tested for this capacity by feeding on them ticks infected with spirochaetae. Two experiments were carried out, in the course of which *Ornithodorus papillipes* ticks, infected with spirochaetae of the tick relapsing fever of the strain «Stalinabad», were being fed on the test-animals. One of the latter (No. 5) perished after 12 days of feeding; we failed, however, to detect spirochaetae in its blood. The other test-animal (No. 6) displayed an open infection, and we were able to observe in its blood the presence of spirochaetae during 9 days, after which we ceased to take blood for examination. Within this period a very great number of spirochaetae was noted on the 5th and 7th day upon the inoculation. In the case the animals were infected by feeding on them ticks, the period of incubation was longer (7 days) than in the case of direct inoculation with infected blood. In the former case the general intensity of the spirochaetosis was lower than in the latter.

The sensitivity of *Dyromys nitedulus* Pall. to the infection with tick relapsing fever when infected tick-transmitters are fed on it gives us reason to believe that

with Spirochaetae of Tick Relapsing Fever

examination

								January 1940																
23	25	26	27	28	29	30	31	2	3	4	5	7	8	9	10	11	13	14	15	16	17	19	20	
+ v. m.	+	+	~	++	+	—	+ v. m.	+ v. m.	+ v. m.	+ v. m.	+ v. m.	++	—	—	—	—	+ v. m.	—	—	—	—			
+ m. v.	+ m.	+ m.	++	+	+	+ v. m.	—	—	—	—	—	—	—	—	—	—	perished							
+	+ m.	++	+ v. m.	+ v. m.	+ m.	++	++	—	—	—	—	perished												
+	+	+	+ m.	—	—	+ v. m.	+ v. m.	perished																
									—	—	—	—	—	—	—	—	perished							
									—	—	—	—	+	+	+	+	+	+ v. m.	+ v. m.	+	+			

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